Better together: Unifying Datalog and Equality Saturation

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Problem: we want everything

In Term Rewriting with EqSat

+ Fast equational reasoning
- Poor analysis support

In Program Analysis with Datalog

+ Composable program analyses
- Quadratic equational reasoning

Can we build one system that subsumes both?

Yes! But How?!

To unify Datalog and EqSat, all you need are

- Functional dependency.
- Functional dependency repair.

Background

EqSat: term rewriting with e-graphs

Big data systems

- Tensor programs [MLSys ‘21, MAPS ‘21]
- Sparse linear algebra [VLDB ‘20]
- Recursive queries [SIGMOD ‘22]

Hardware

- DSP vectorization [ASPLOS ‘23]
- Datapath optimization [ASP-DAC ‘23]

Program optimization

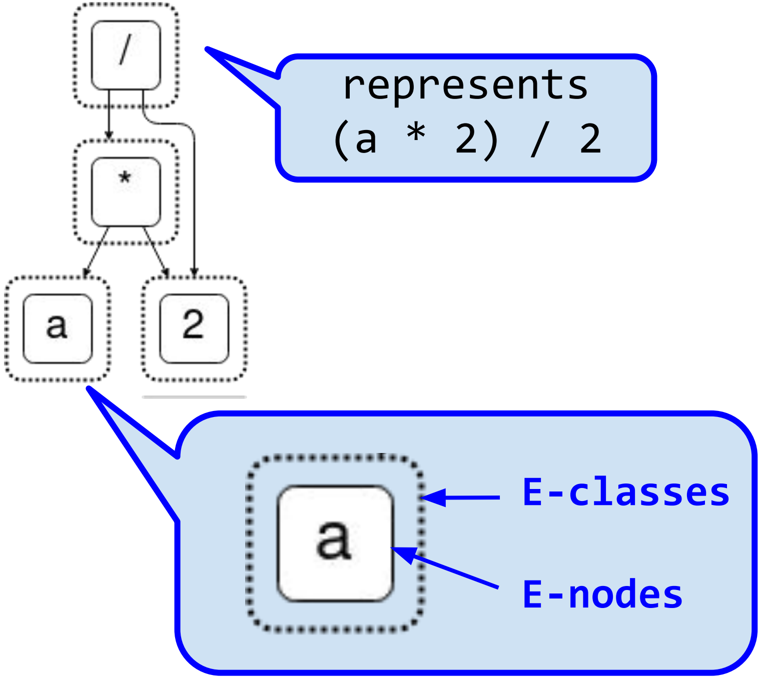
- Imperative programs [POPL ‘09]
- Functional programs [EGRAPHS ‘22]
- Floating-point expression [PLDI ‘15]

Program synthesis

- CAD parametrization [PLDI ‘20]
- Rewrite rule synthesis [OOPSLA ‘21]

intel RisingLight An Educational OLAP Database System CERTORA BYTECODE ALLIANCE

E-graphs and Equality saturation



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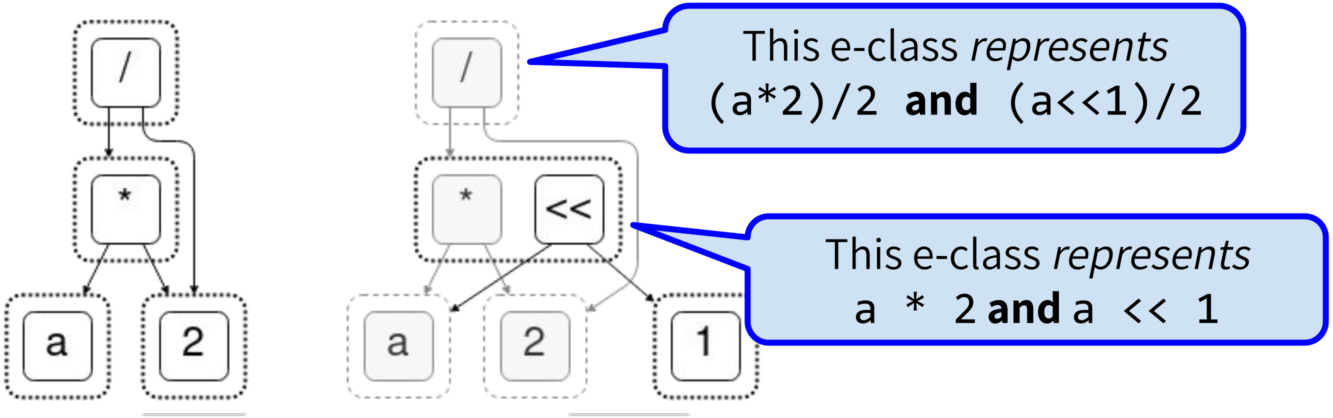

`x * 2 => x << 1`

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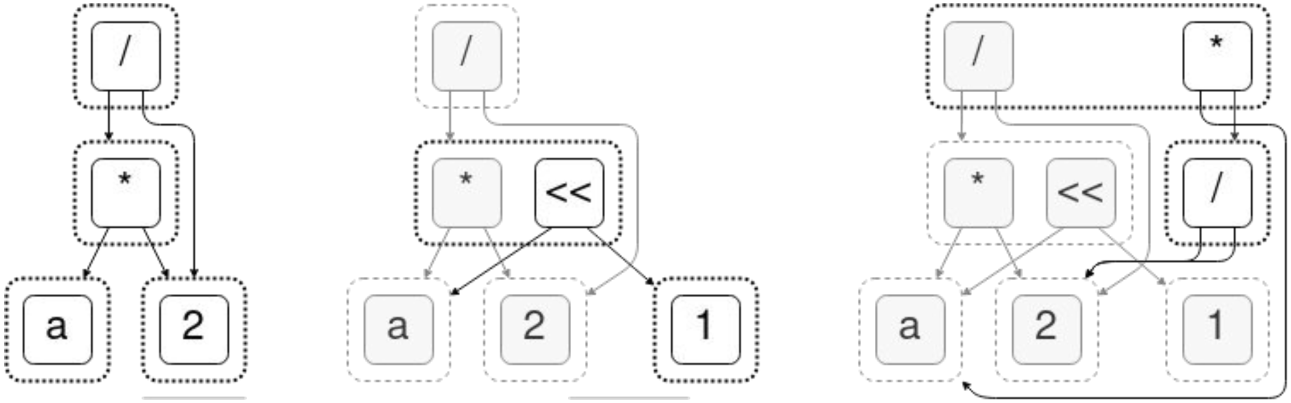

x * 2 => x << 1   (x * y) / z => x * (y / z)

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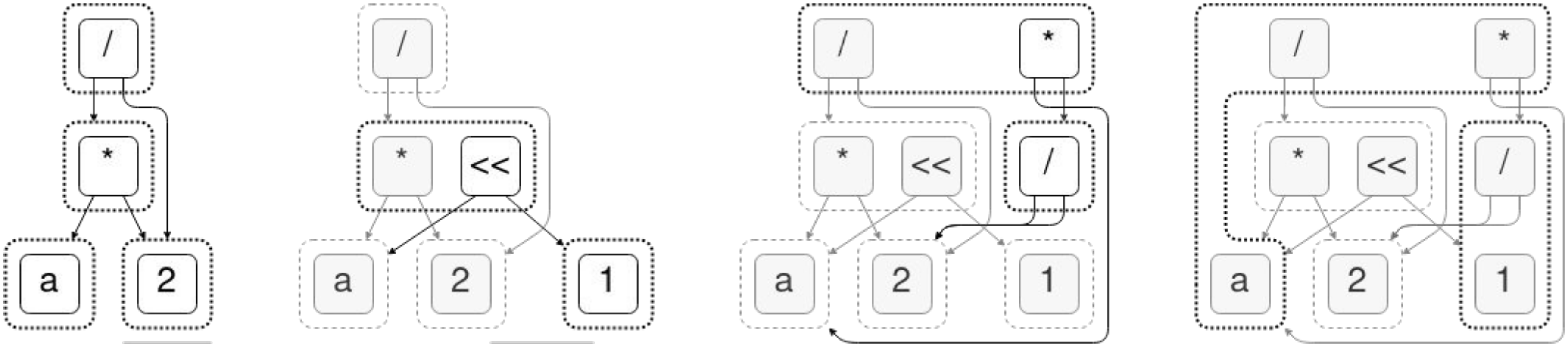


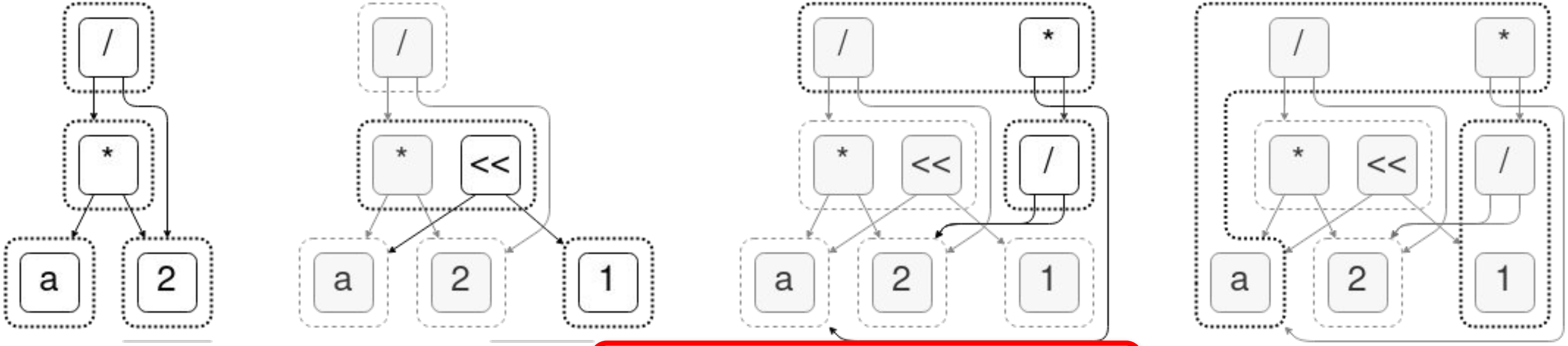
`x * 2 => x << 1`

`(x * y) / z => x * (y / z)`

`x / x => 1`
`x * 1 => x`

loop until
fixpoint / timeout!

E-graphs and Equality saturation



x * 2 => x <<

x has to be non-zero!

x / x => 1

x * 1 => x

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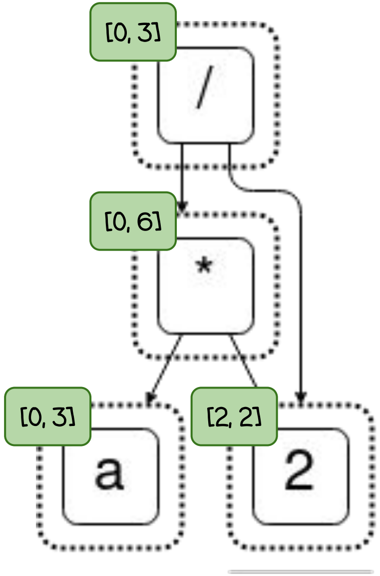
xpoint / timeout!

Needs semantic information here

E-class analyses



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- Each E-class is abstracted w/ a lattice.

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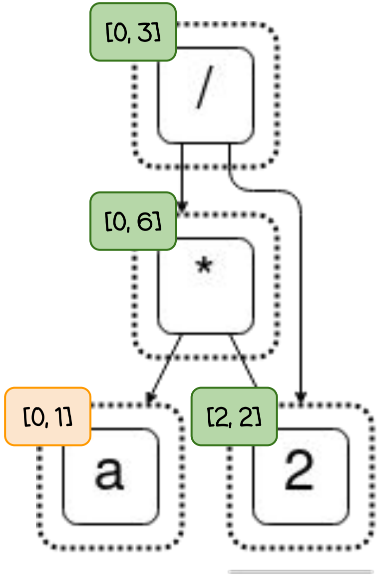

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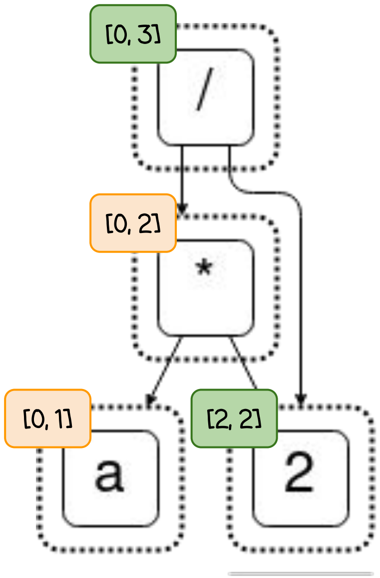

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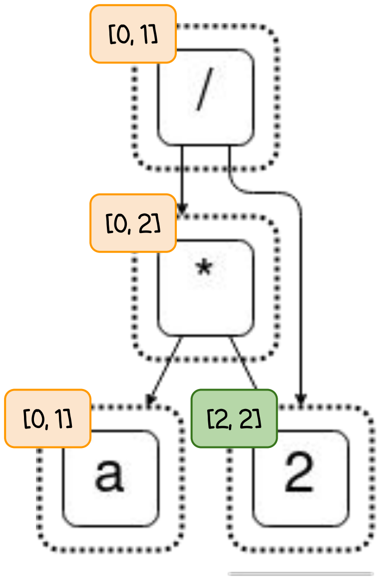

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E-class analyses

[0, 1]

E-class analysis has severe limitations:

- Only one analysis allowed.
- Facts only propagate from children to parents.
  - Type checking 😭
- Monolithic Rust implementation of one big analysis.
  - Not composable!

tice.

BUT: analyses are rules, too!

Program analysis in Datalog

- Multiple analyses ✅
- Modular ✅
- Composable ✅

```
// If expression e is a number,
// its lower bound is itself
num(n, e) ⇒ lower_bound(e, n).

// If expression e has the form x + y,
// its lower bound is the lower bound of x
// plus the lower bound of y.
add(x, y, e) ∧
  lower_bound(x, lx) ∧
  lower_bound(y, ly) ⇒
lower_bound(e, lx + ly).

// If the lower bound of e is greater than 0,
// e is nonzero.
lower_bound(e, le) ∧ le > 0 ⇒
  nonzero(e)
```

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han 0,

Can we do EqSat in Datalog?

We need to express in Datalog

(+ (+ x y) z) ⟶ (+ x (+ y z))

triggers

actions

Relational E-matching (POPL 2022)

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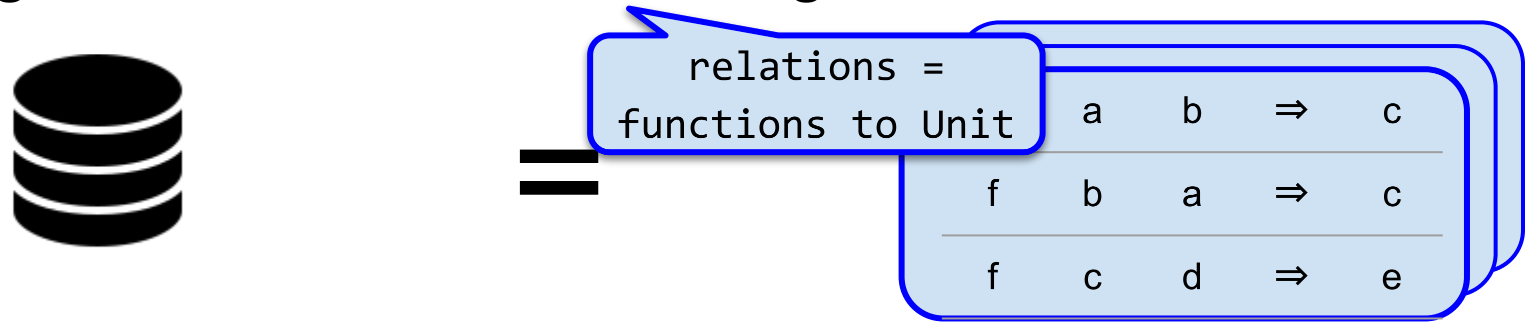
- Significant speedups.
- This handles triggers ✅
- *What about actions?*

`egglog`: unifying Datalog and EqSat

egglog's key concept: functions

Using a function-first database design



Database

Set of function tables

egglog's key concept: functions

Using a function-first database design

Database

=

f				
	a	b	⇒	c
	b	a	⇒	c
	c	d	⇒	e

Set of function tables

Now we can talk about

- terms like `f(f(a, b), d)` and
- equivalences like `f(a, b)` = `f(b, a)`

Equality saturation in egglog

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(datatype Math (Num i64)
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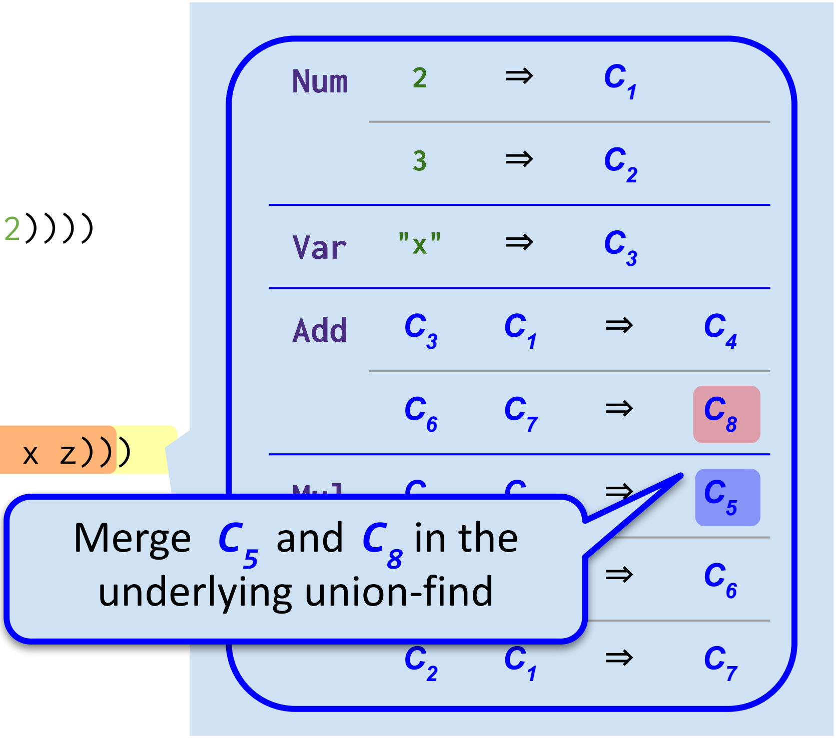

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Key idea: functional dependency repair

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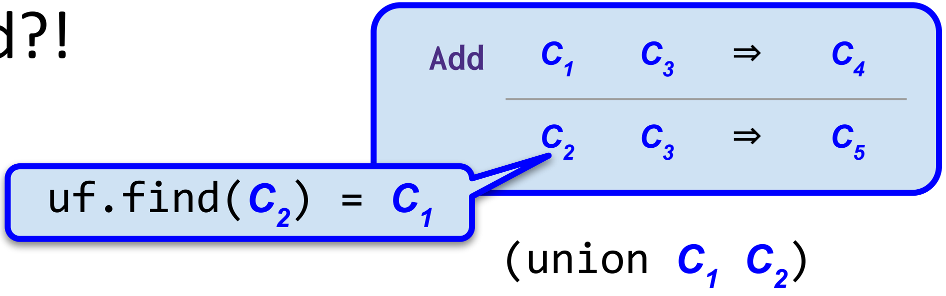

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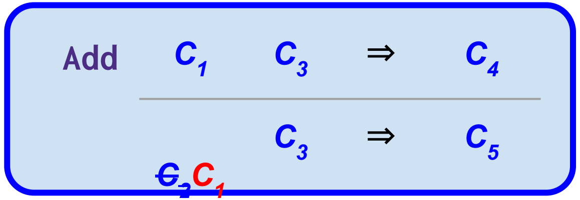

(union c_1 c_2)

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Add C_1 C_3 ⇒ $\{C_4, C_5\}$

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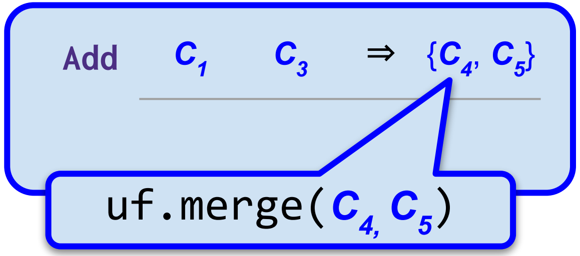
- We merge the conflicting values with a union find!



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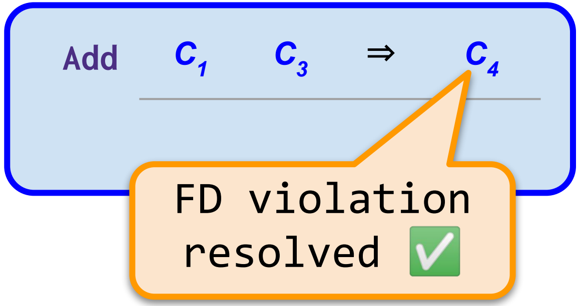
- What if this is violated?!
- We merge the conflicting values with a union find!



Key idea: functional dependency repair

The same mechanism also enables **composable** analyses.

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(function hi (Math) Rational)

(function lo (Math) Rational)
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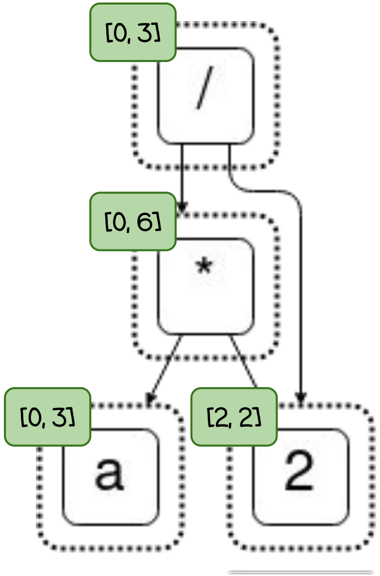

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(function hi (Math) Rational :merge (min old new))

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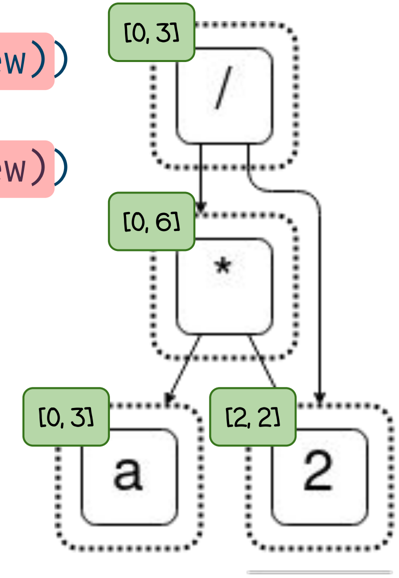

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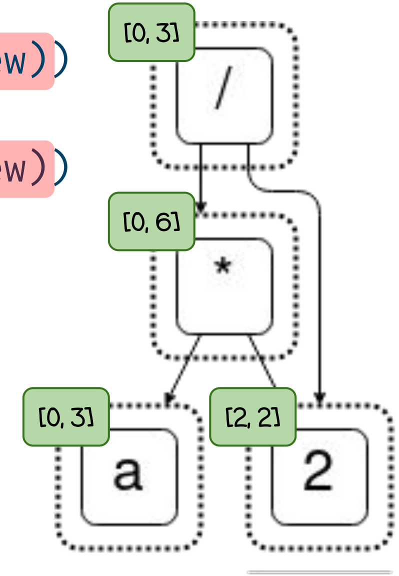
hi	C_1	⇒	3
	C_2	⇒	2

(union C_1 C_2)



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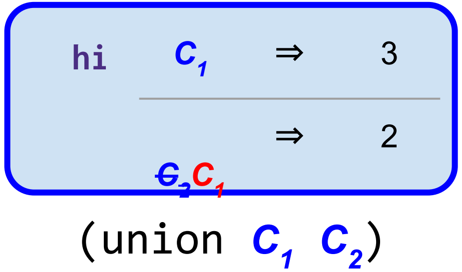

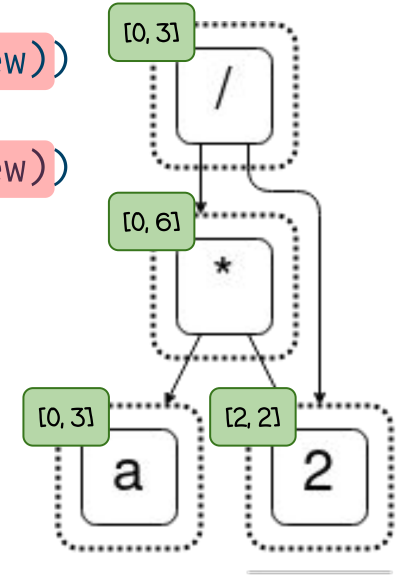

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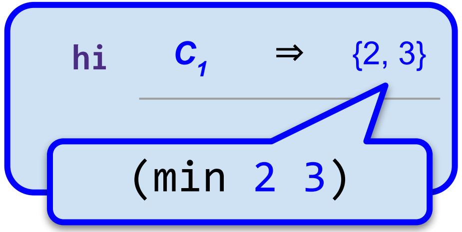

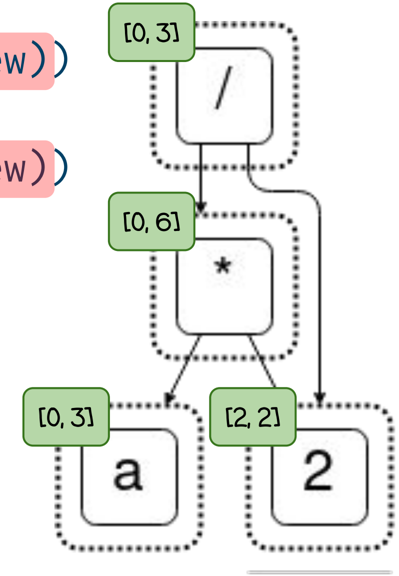

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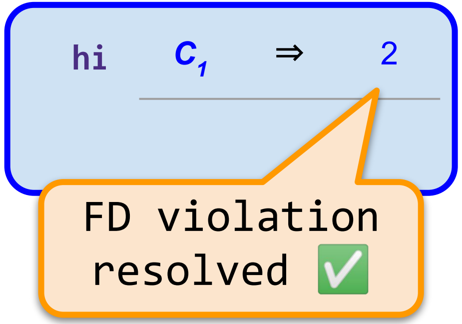
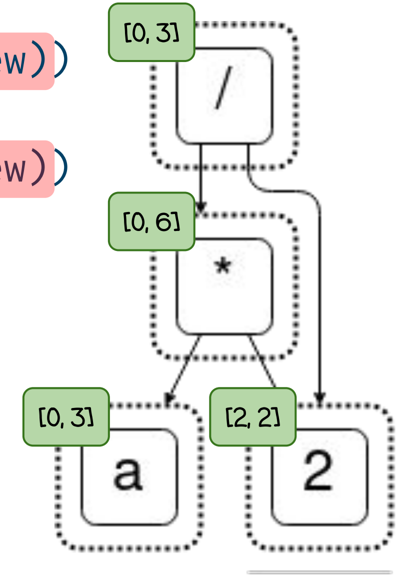




Evaluation

Enabling new optimizations

Database-like architecture

- Relational e-matching.
- Efficient query evaluation.

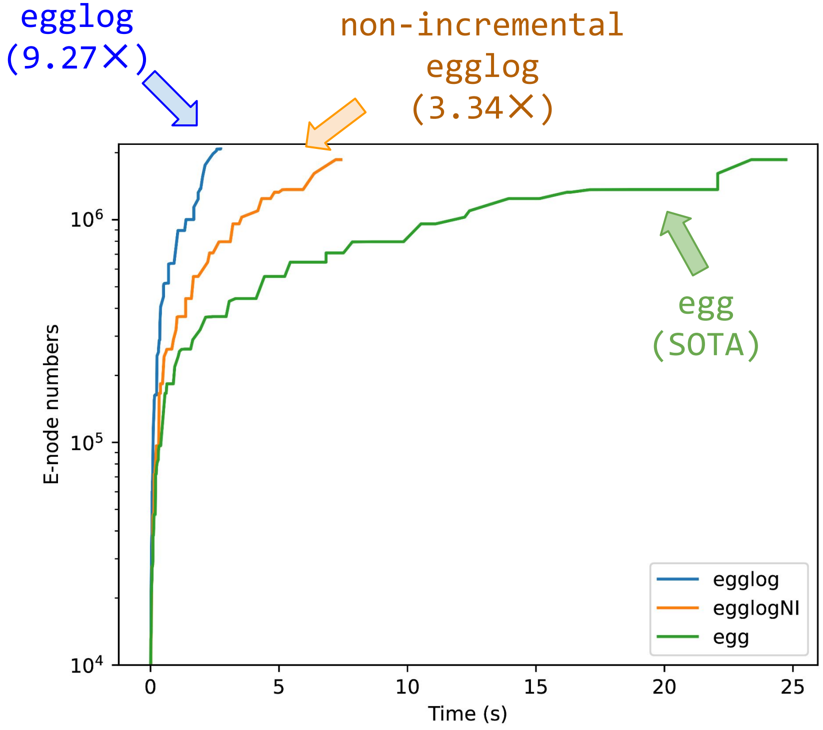
Enabling new optimizations

Database-like architecture

- Relational e-matching.
- Efficient query evaluation.

Incrementalization

- Incremental EqSat is hard.
- We use the standard semi-naive evaluation of Datalog to make EqSat incremental for free.



math micro-benchmark

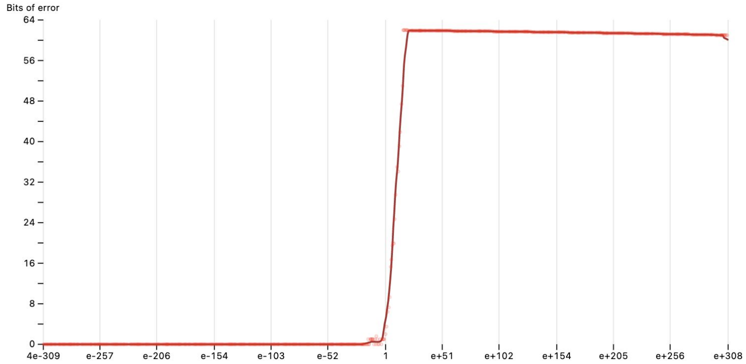
Herbie

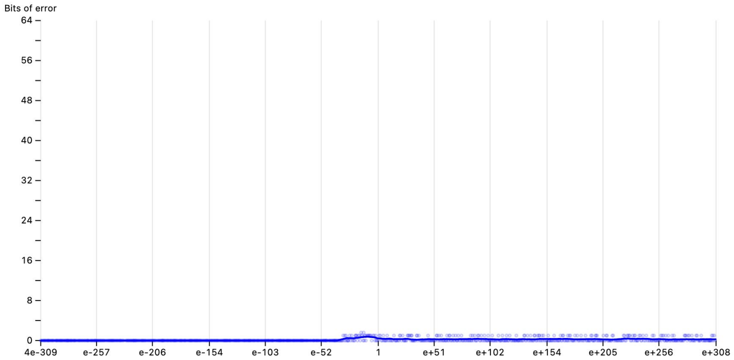
$$\sqrt{x+1} - \sqrt{x}$$

$\Rightarrow$

$$\frac{1}{\sqrt{1+x} + \sqrt{x}}$$

less floating-point errors,
more accurate!

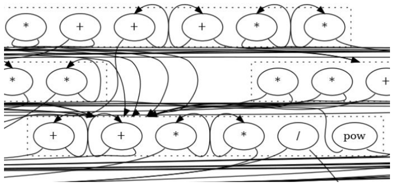




Herbie

$$\sqrt{x+1} - \sqrt{x}$$

EqSat



Extract

$$\frac{1}{\sqrt{1+x} + \sqrt{x}}$$

```
x + y ⇒ y + x
(x + y) + z ⇒ x + (y + z)
```

$\sin^2 x + \cos^2 x \Rightarrow 1$

$x^{yz} \Rightarrow x^y \cdot x^z$

$x^{-1} \Rightarrow 1 / x$

…

Not all identities can be expressed as syntactic rules.

$x / x \Rightarrow 1$ **when** $x \neq 0$

$(x \cdot y) / z \Rightarrow x / (z / y)$
when $y \neq 0$

$x - y \Rightarrow (x^3 - y^3) / (x^2 + xy + y^2)$
when $x \neq 0$ **or** $y \neq 0$

…

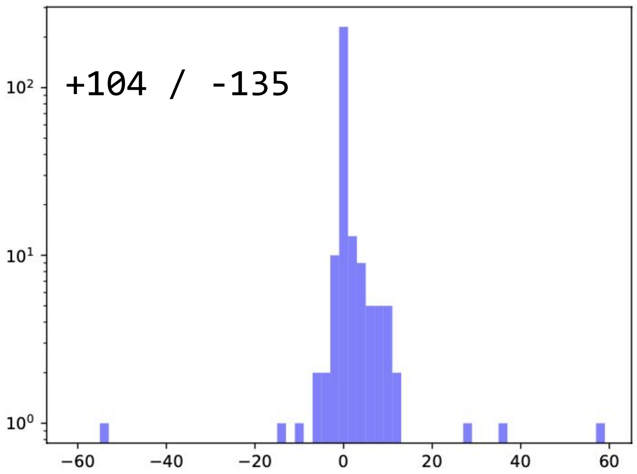
Herbie

When unsoundness is detected, Herbie has to discard the results and roll back 😭

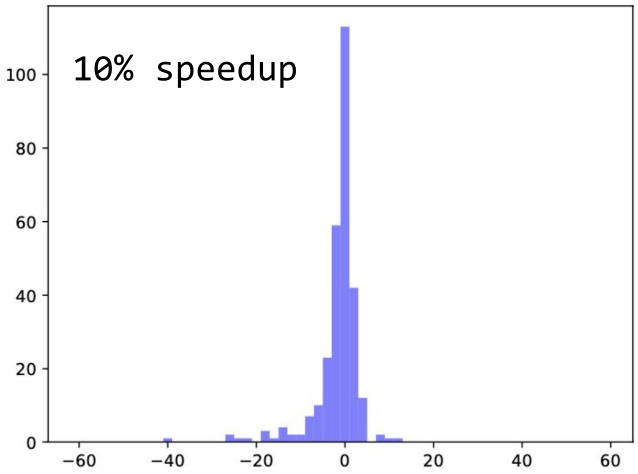
We make Herbie's rules sound with E-graph program analyses in egglog:

- Interval analysis
- Definability analysis

Results on Herbie's benchmark



Accuracy ("# bits")



Time (s)

Results on Herbie's benchmark

Our reimplementation in egglog achieves a comparable accuracy and performance, but does not suffer from the soundness issue in original Herbie.

Herbie's design made simpler ✅

Bringing the power of unification to Datalog

Datalog is good at program reasoning tasks such as

- Pointer analyses.
- Type checking/inference

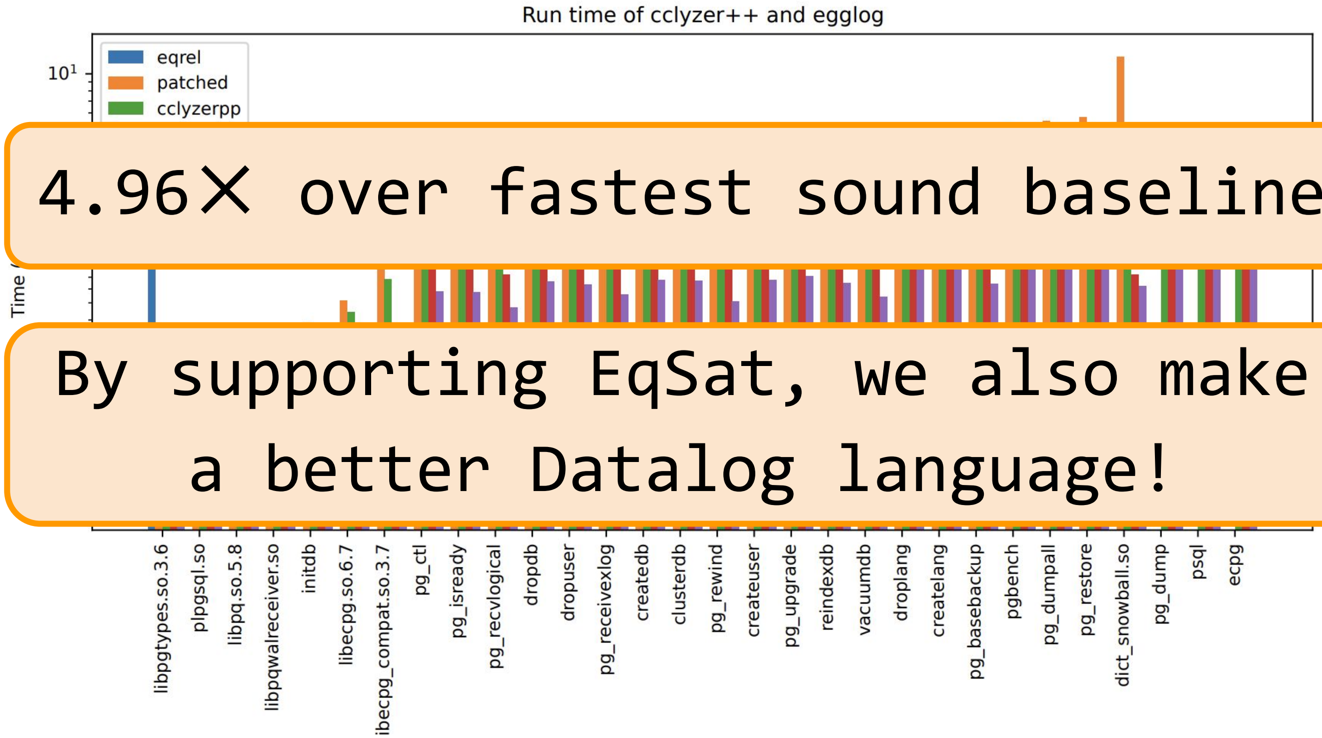
However, many advanced program reasoning tasks also require equivalence reasoning

- Steensgaard pointer analysis.
- Hindley-Milner type inference.

`Datalog:` 😟

`egglog:` ☺️

Steensgaard-style points-to analysis



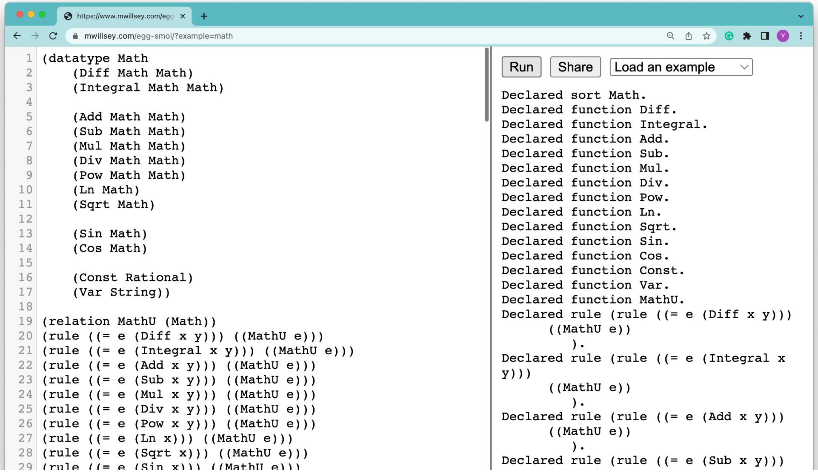
4.96× over fastest sound baseline

By supporting EqSat, we also make a better Datalog language!

egglog: Unifying Datalog and Equality Saturation

By unifying Datalog and EqSat, we get

- ✅ Fast equational reasoning *a la* EqSat.
- ✅ Rich composable analyses *a la* Datalog.
- ✅ Fast and incremental eval with DB magic.
- ✅ User-friendly language interface.



egraphs-good.github.io/egglog

Thank you

Remy Wang

Oliver Flatt

David Cao

Philip Zucker

Eli Rosenthal

Zach Tatlock

Max Willsey

Key idea: functional dependency repair

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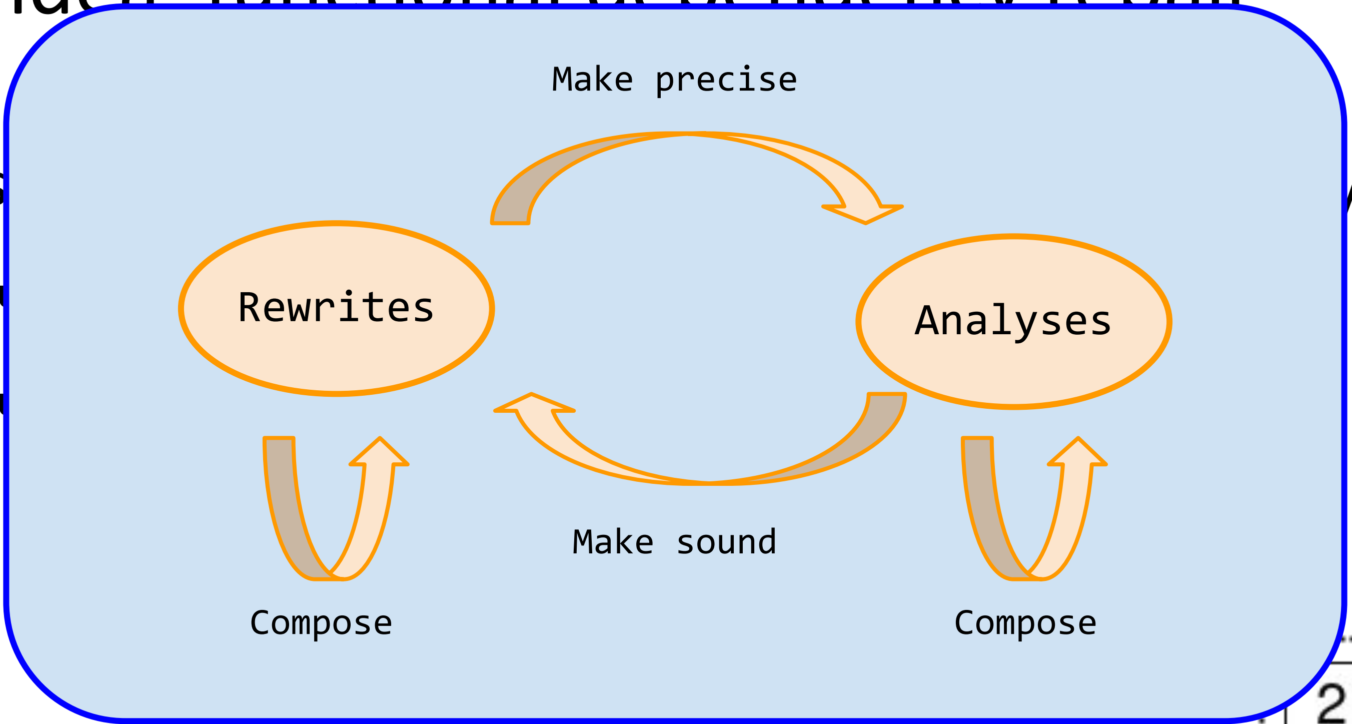

resolved

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