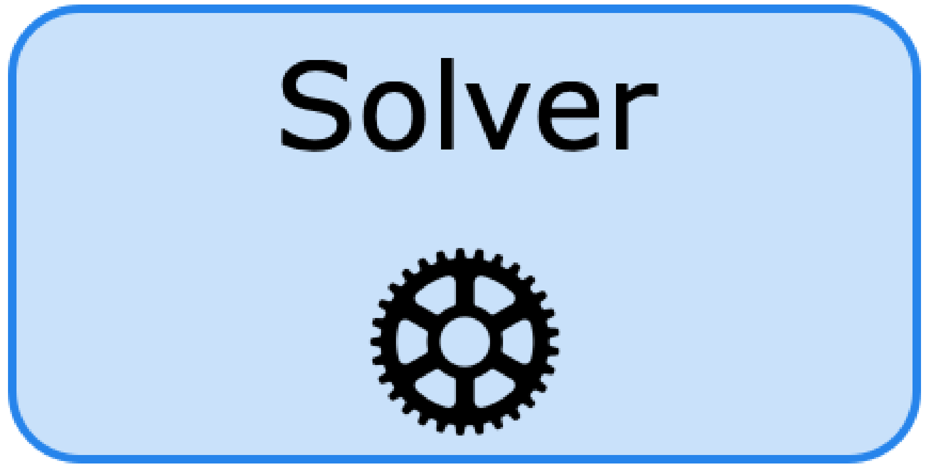
Small Proofs from Congruence Closure

Oliver Flatt, Samuel Coward, Max Willsey,

Zachary Tatlock, Pavel Panchekha

Solvers and Proofs



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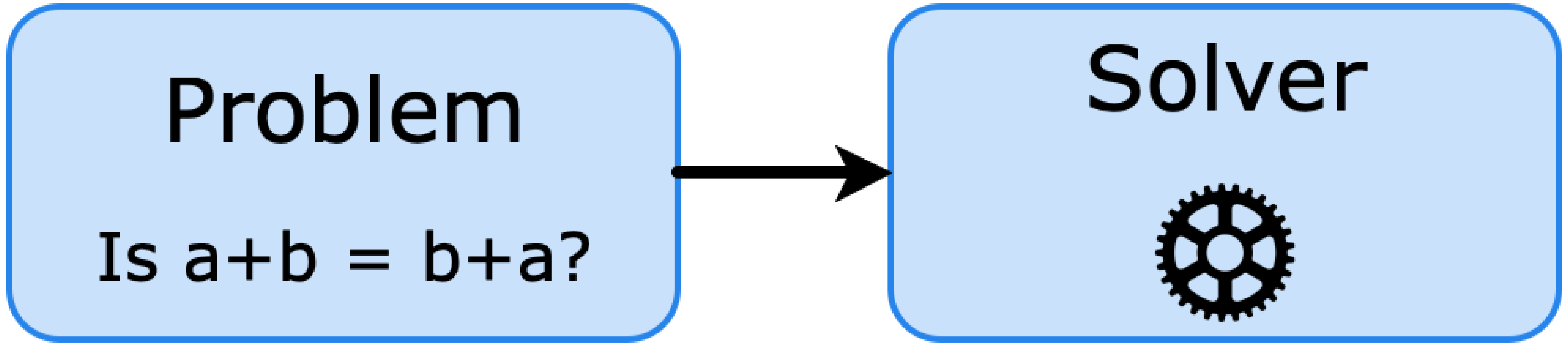

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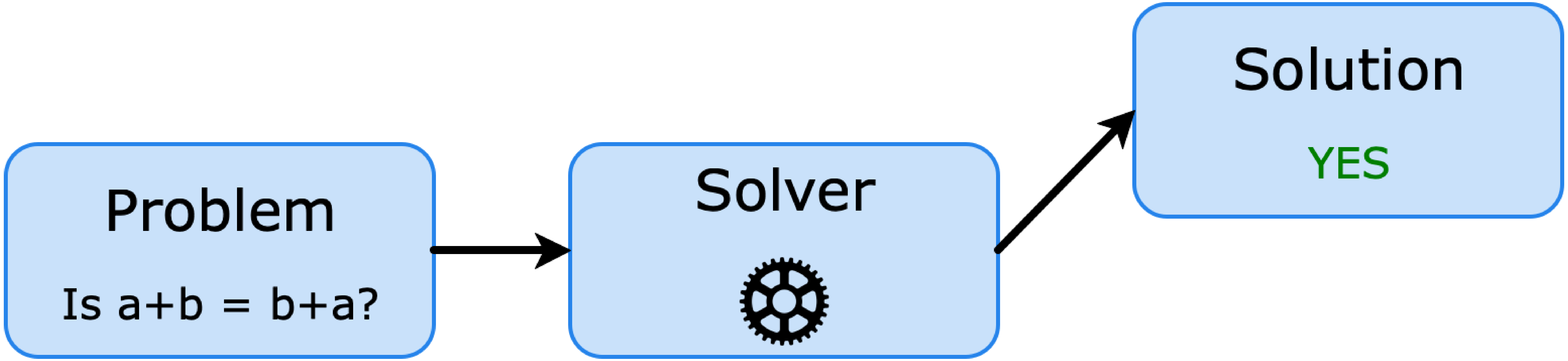

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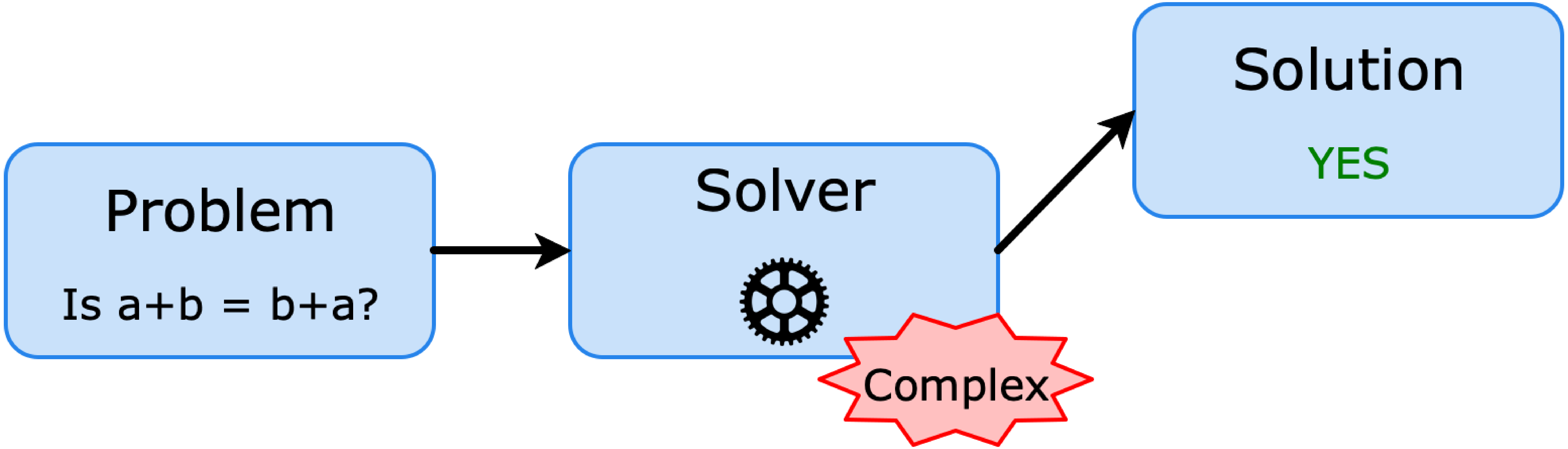

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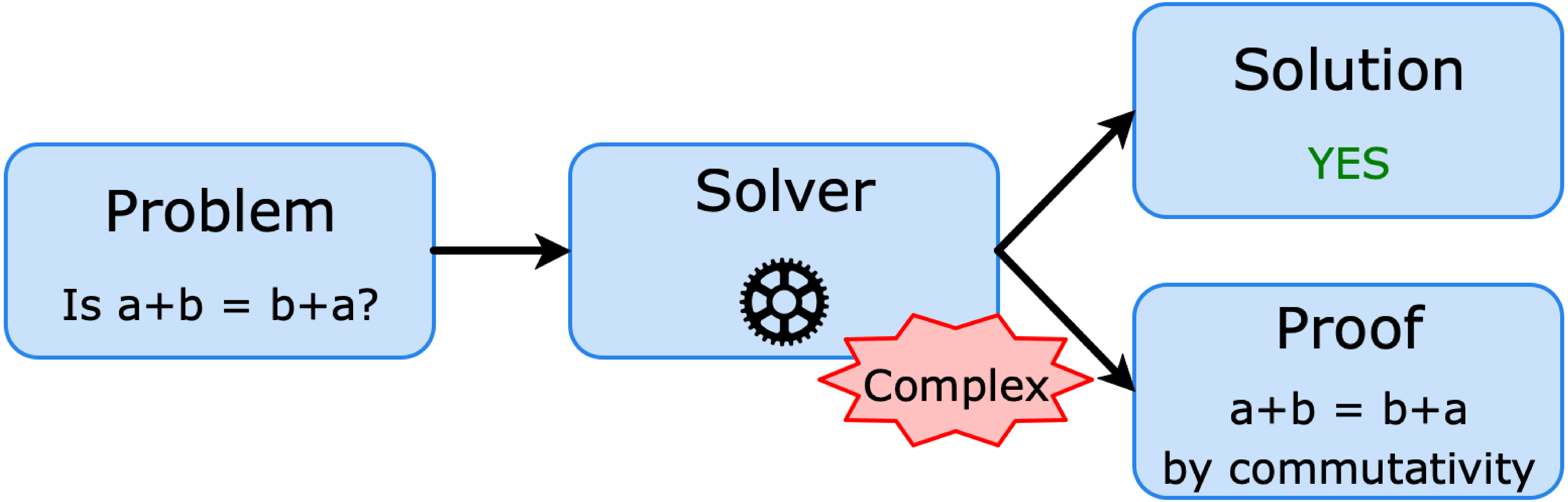

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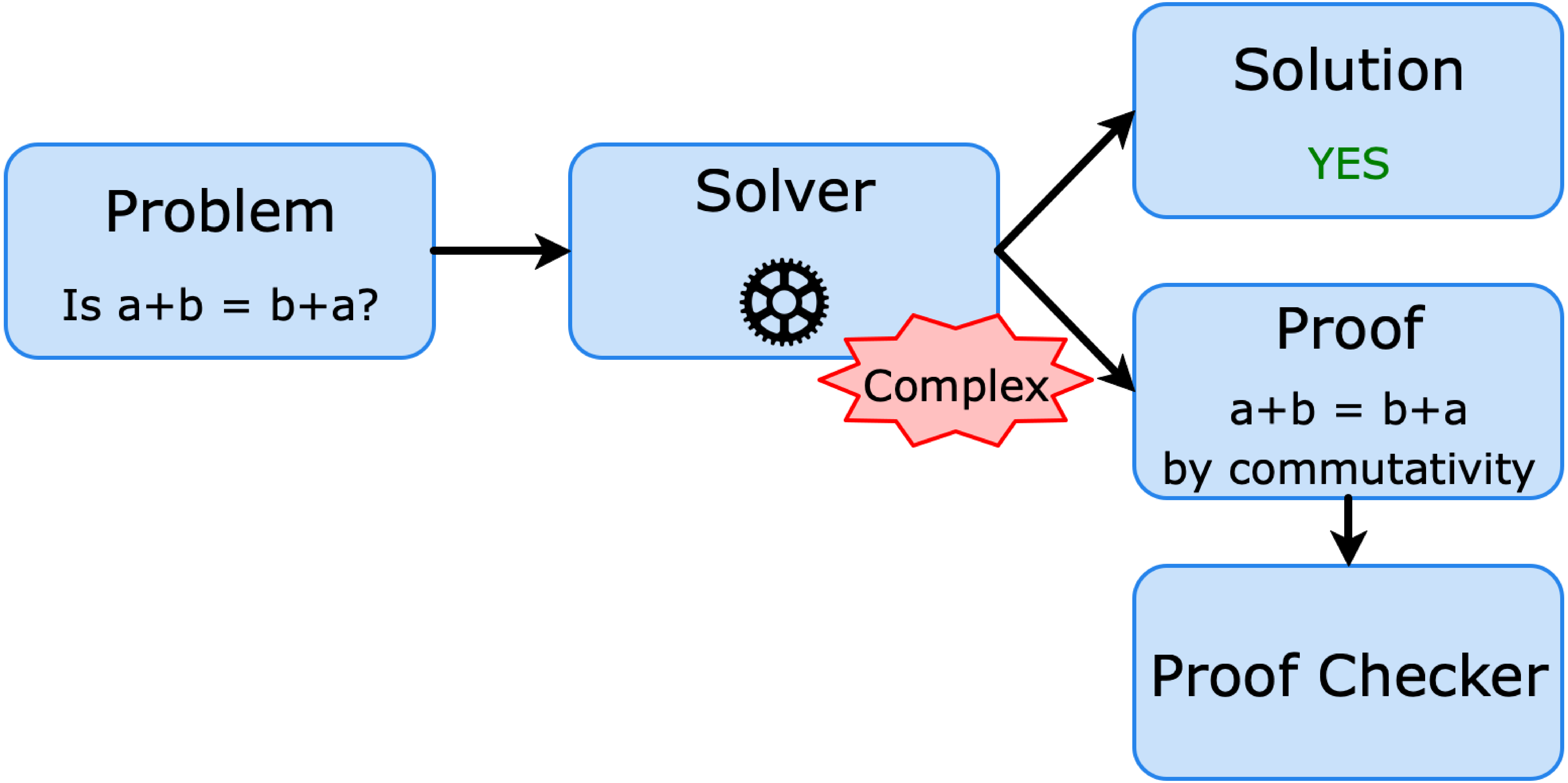

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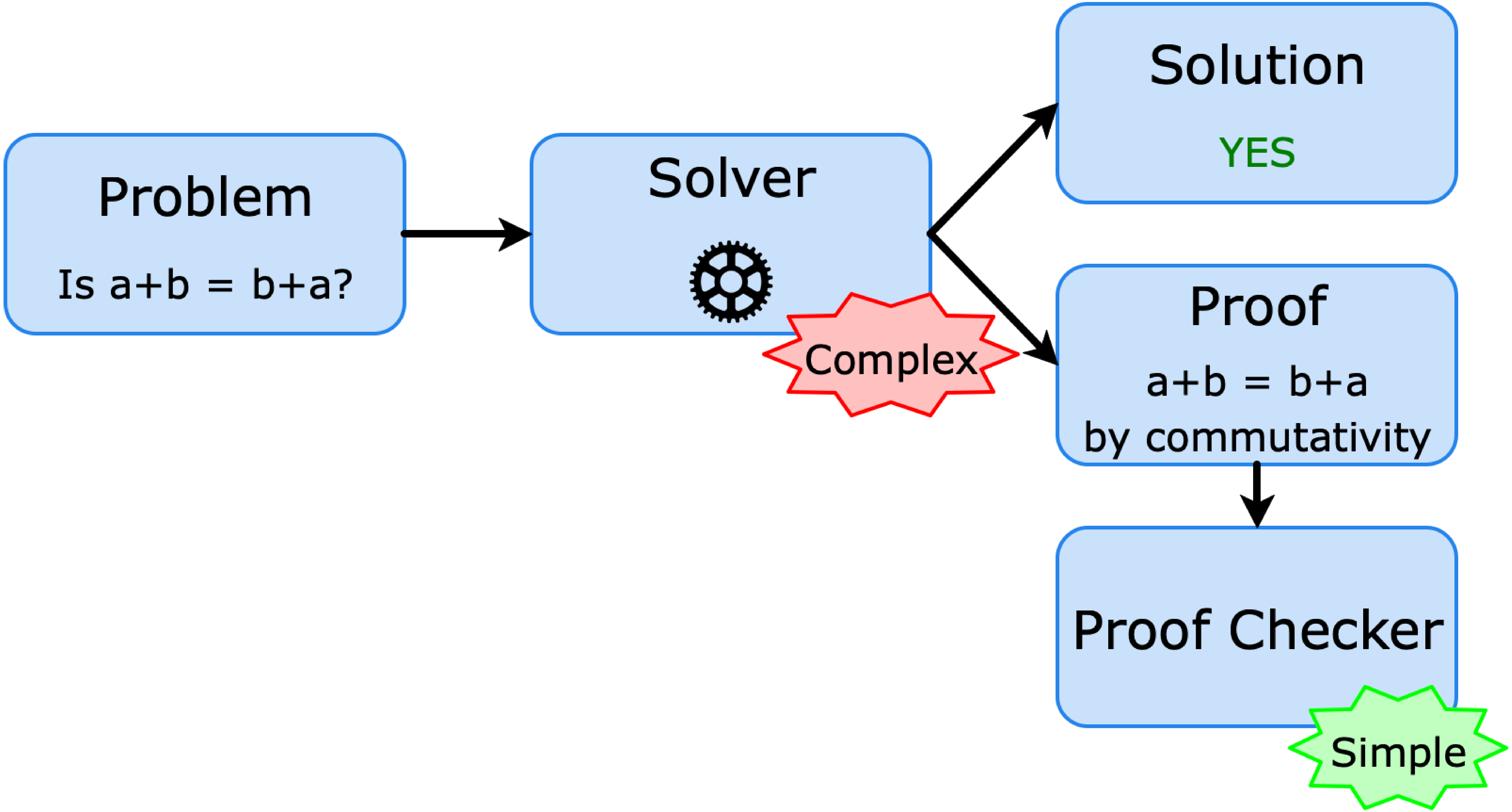

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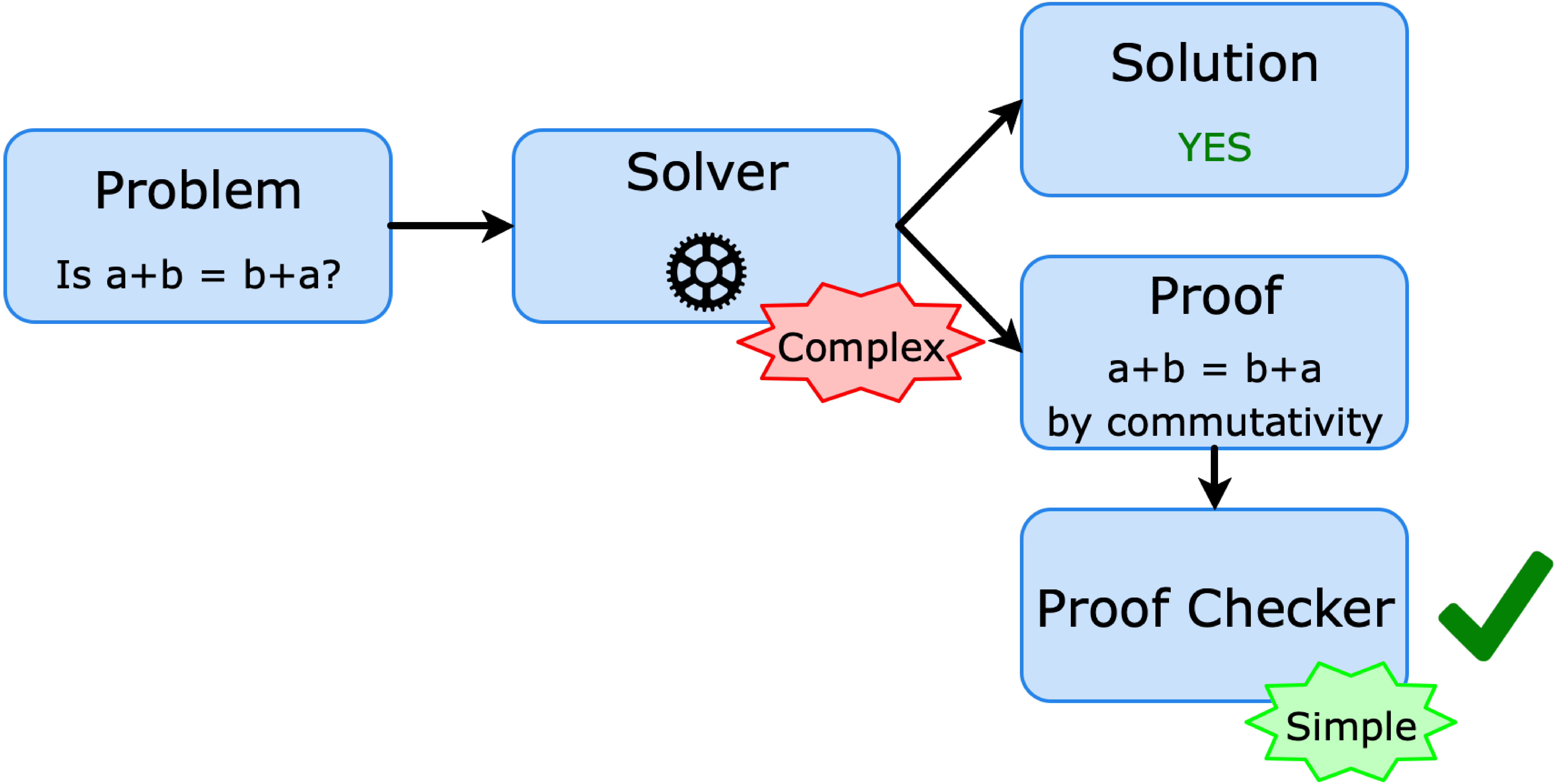

Proofs are Useful

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Checking

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Checking

Can we trust the solver?

Proofs are Useful

Checking

Can we trust the solver?

Debugging

Proofs are Useful

Checking

Can we trust the solver?

Debugging

How did we prove $0 = 1$?

Proofs are Useful

Checking

Can we trust the solver?

Debugging

How did we prove 0 = 1?

CDCL

Proofs are Useful

Checking

Can we trust the solver?

Debugging

How did we prove 0 = 1?

CDCL

What facts led to this result?

Proofs are Useful

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What facts led to this result?

...And More

Fuzzing

Optimization

Proofs can be Long

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Debugging

Confusing

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Too Specific

...And More

Fuzzing

Optimization

Proofs can be Long

Checking

Okay

Debugging

Confusing

CDCL

Too Specific

...And More

Slow

Proofs can be Long

This Talk:

Finding **smaller** proofs from congruence closure

Why Congruence Closure?

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Congruence Closure forms the basis of many solvers

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Generates all proofs of **equality**

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Enables **equality saturation**

Optimization and synthesis

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- Optimization and synthesis
- Our library: egg

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- [x] Motivation
- [ ] Congruence Closure
- [ ] Proofs from Congruence Closure
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27.2% smaller proofs!

Congruence Closure

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Input: equalities between terms

a = b

f(a) = f(b)

b = c

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Output: equivalence relation

stored in an **e-graph** data structure

Ask: is a = c?

Congruence Closure

Input: equalities between terms

$a = b$

$f(a) = f(b)$

$b = c$

Output: equivalence relation

stored in an **e-graph** data structure

Ask: is $a = c$?

The relation is also closed under **congruence**

$\forall\ x, y: x = y \Rightarrow f(x) = f(y)$

E-Graph Example

A graph with **3** kinds of edges

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Inputs:

f(a)

f(b)

a = b

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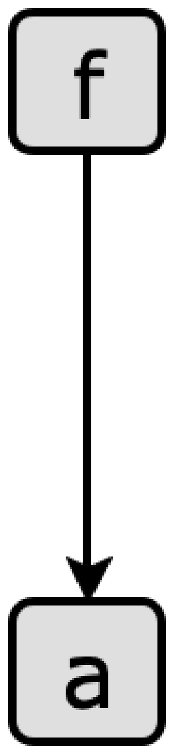

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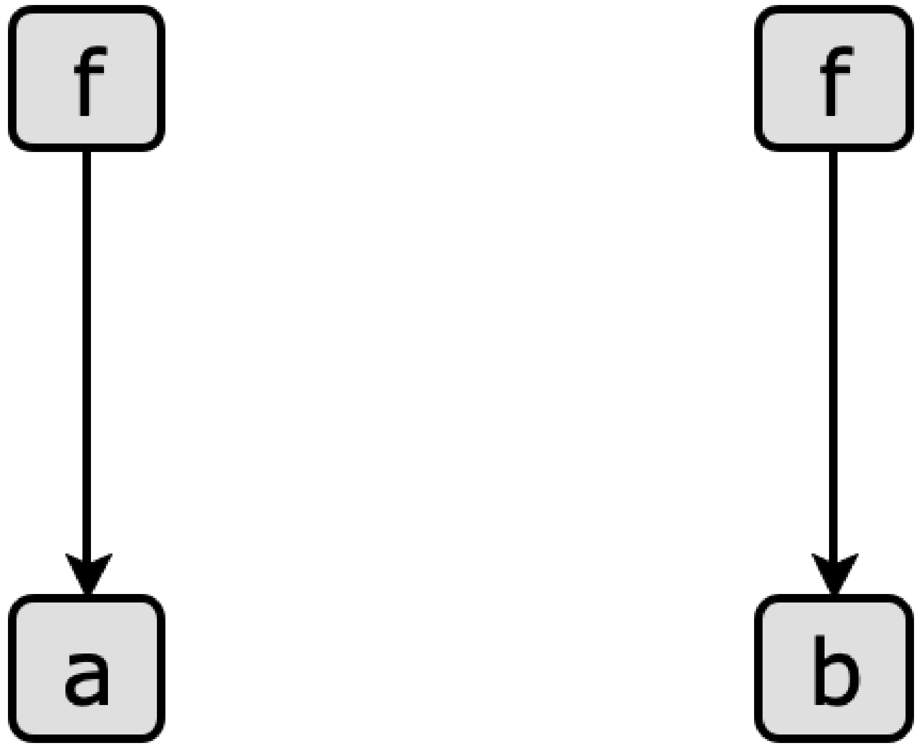

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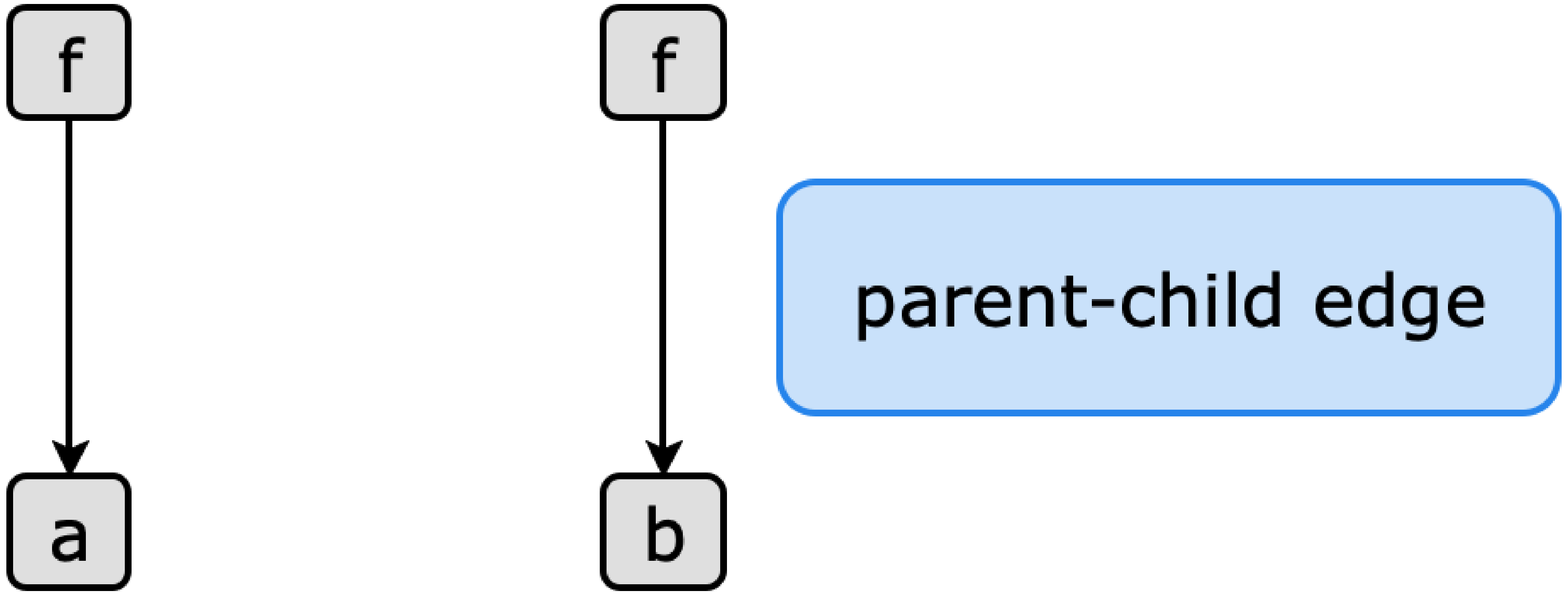

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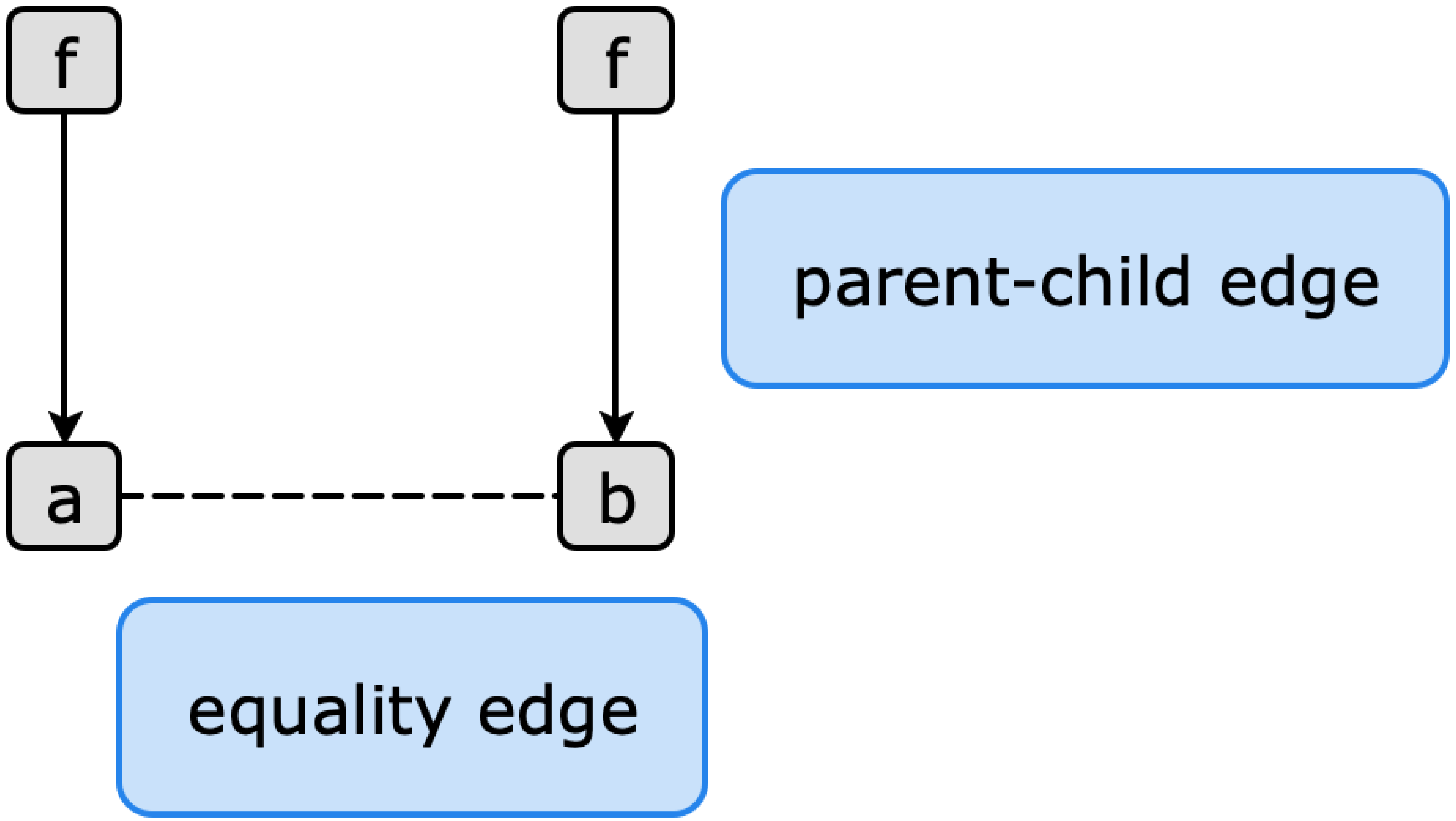

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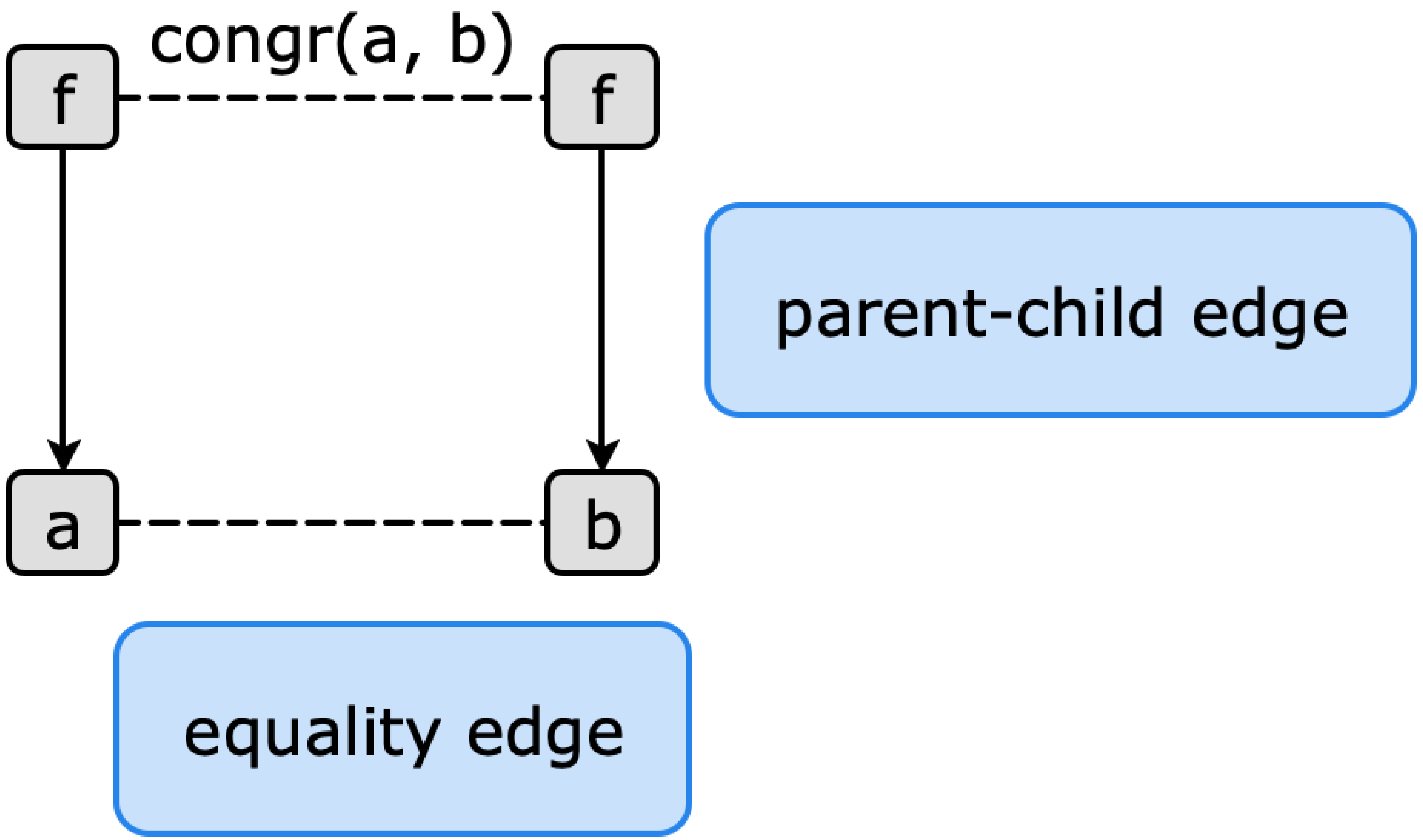

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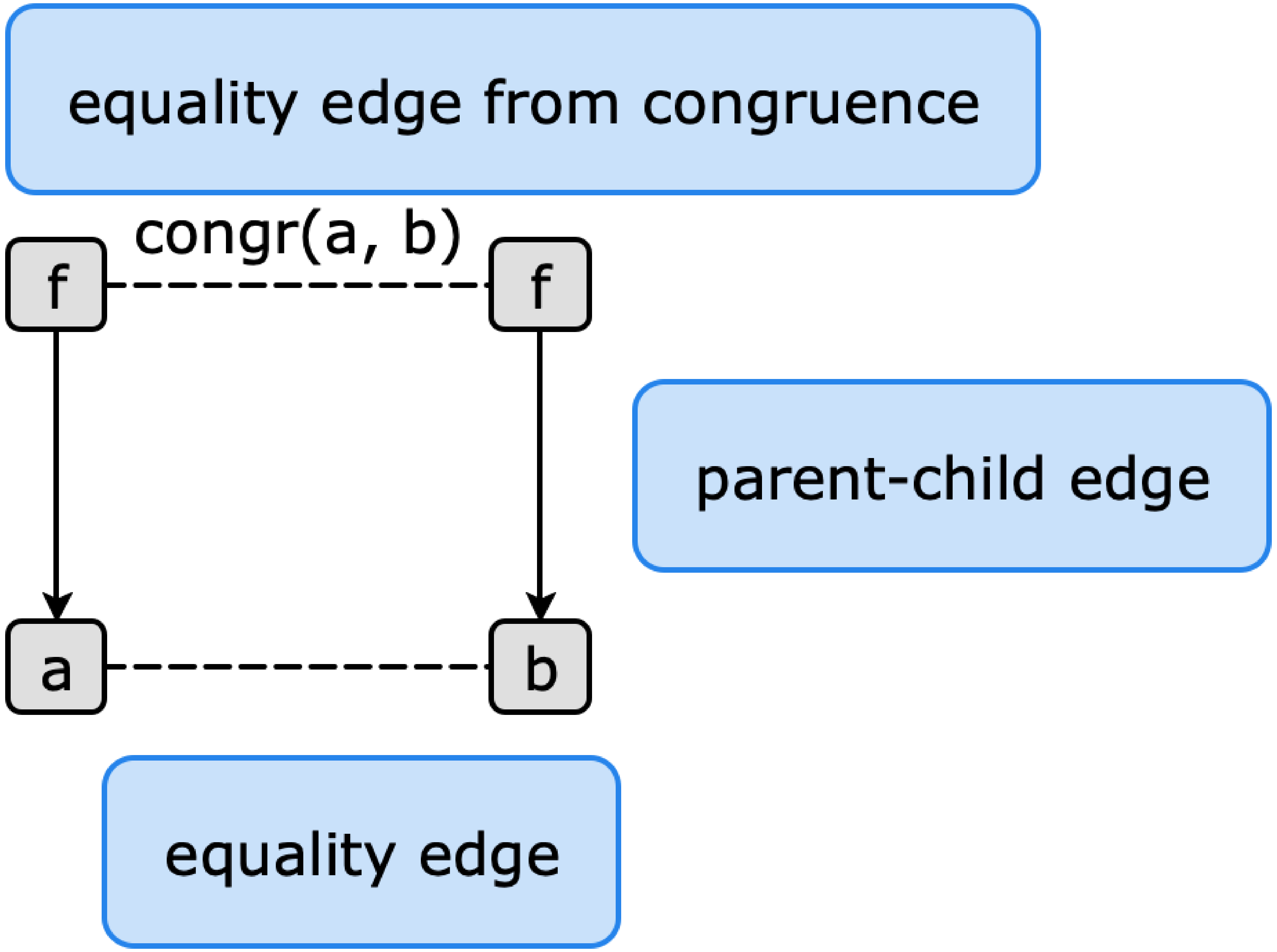

A Bigger E-Graph Example

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Inputs:

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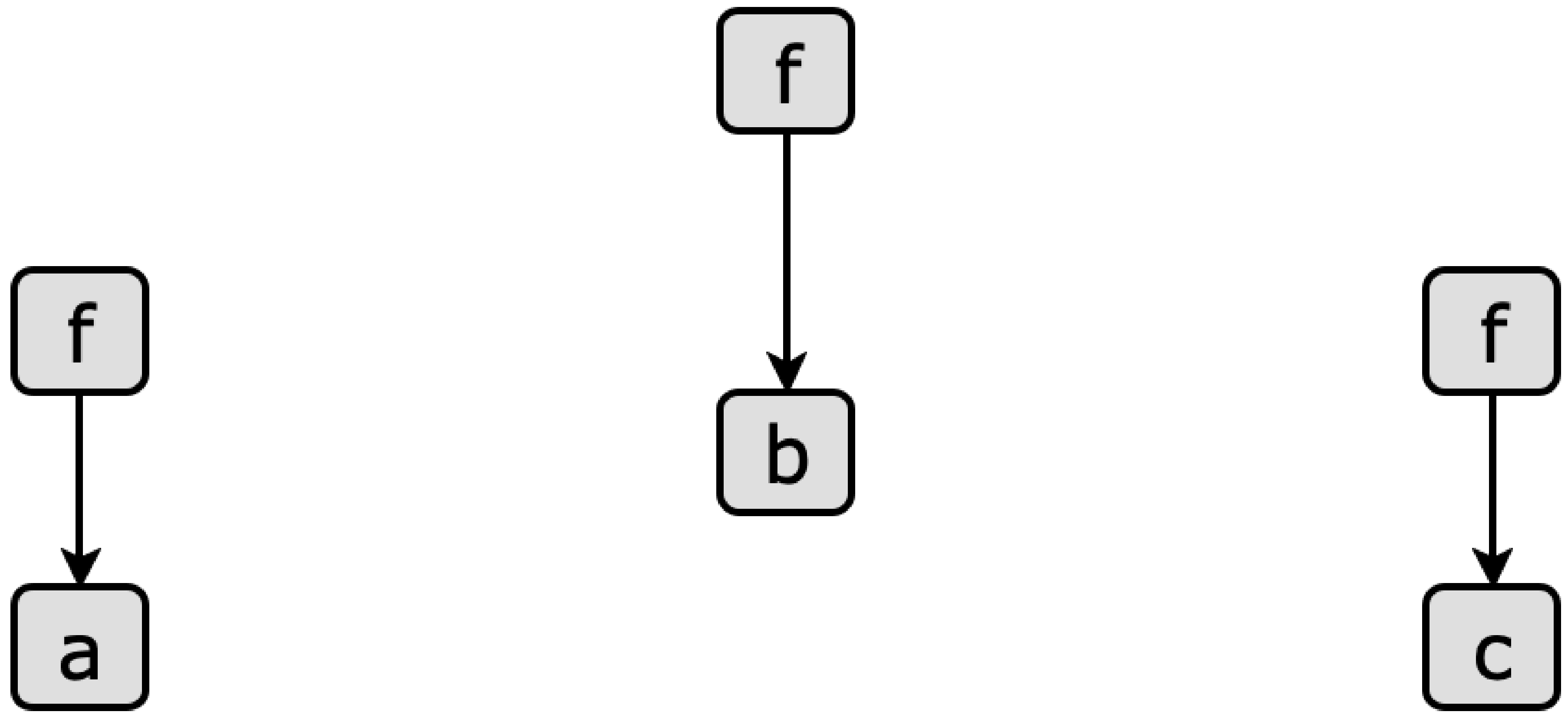

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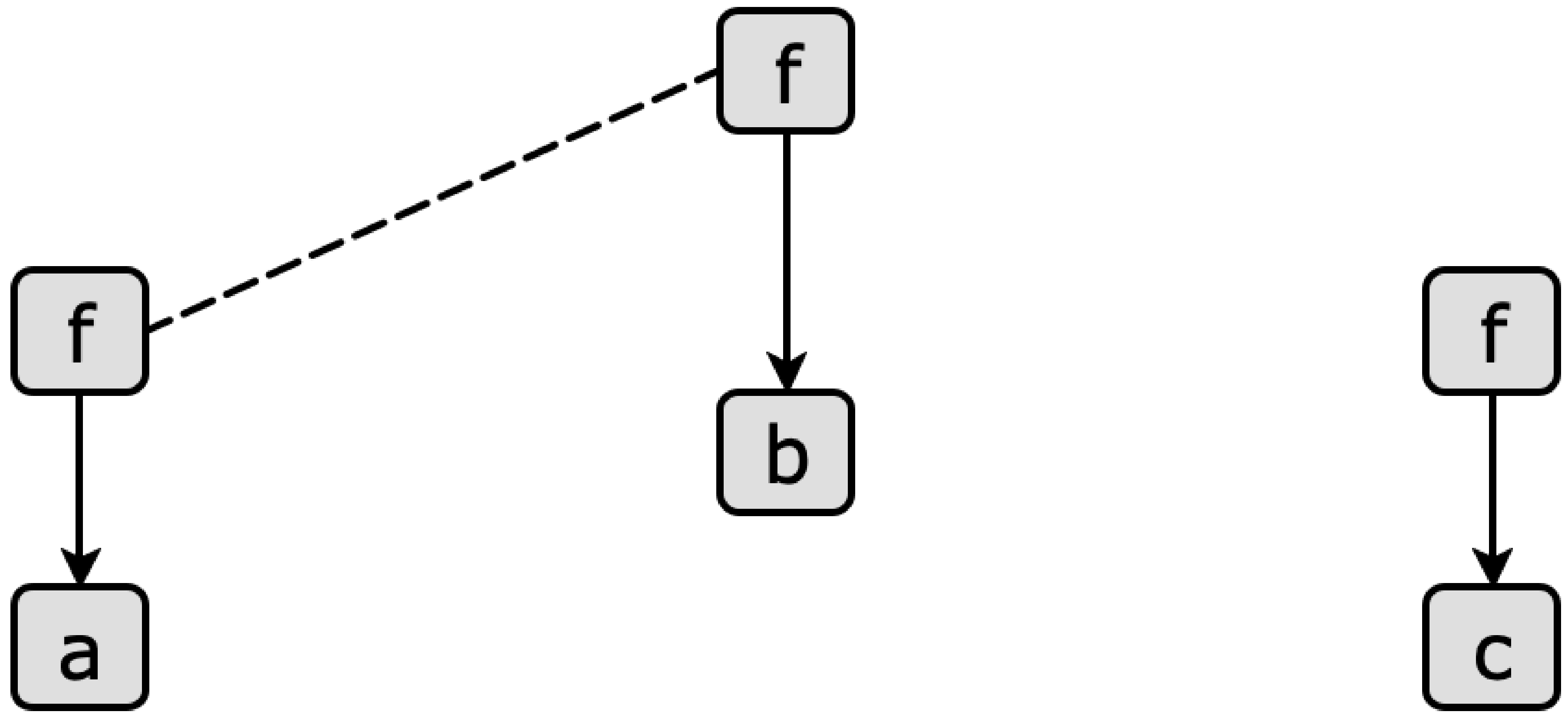

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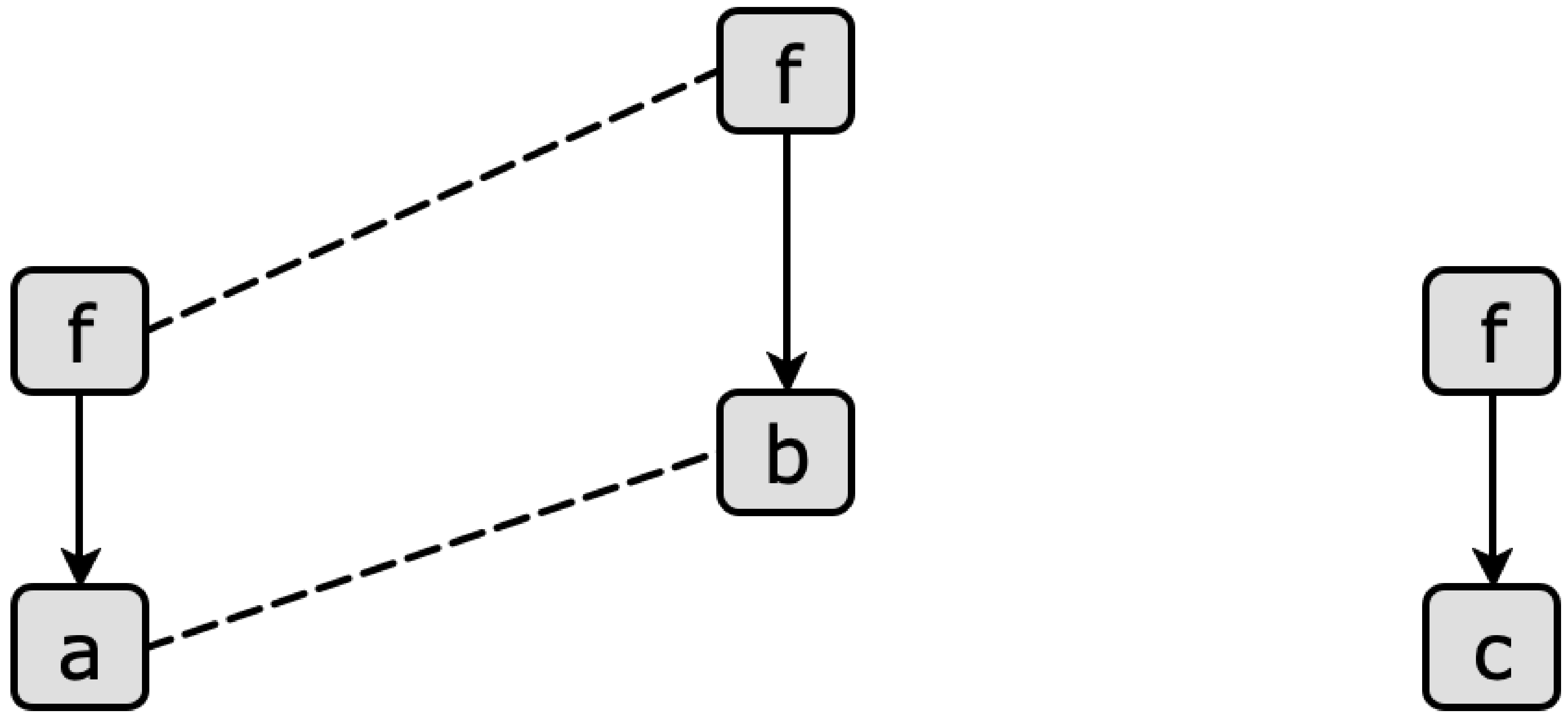

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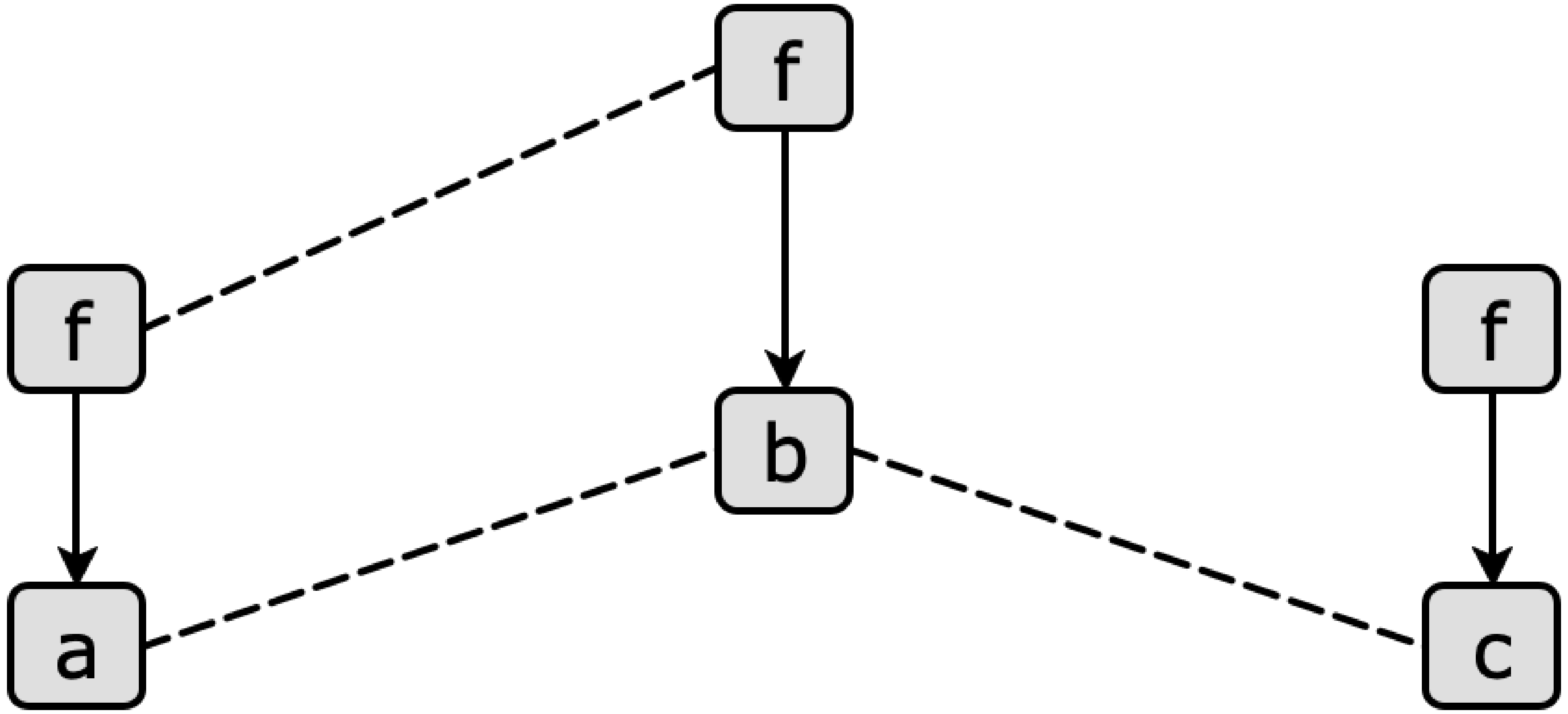

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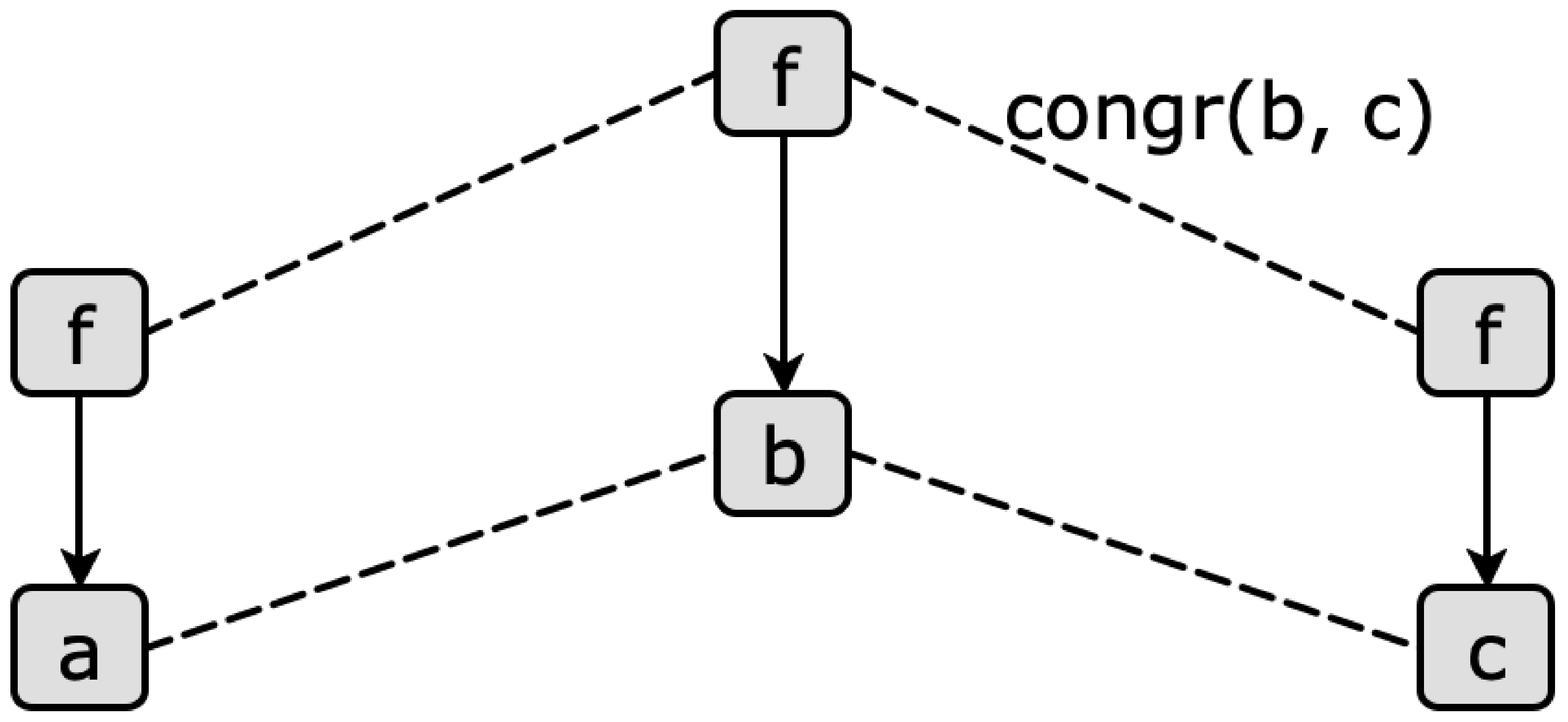

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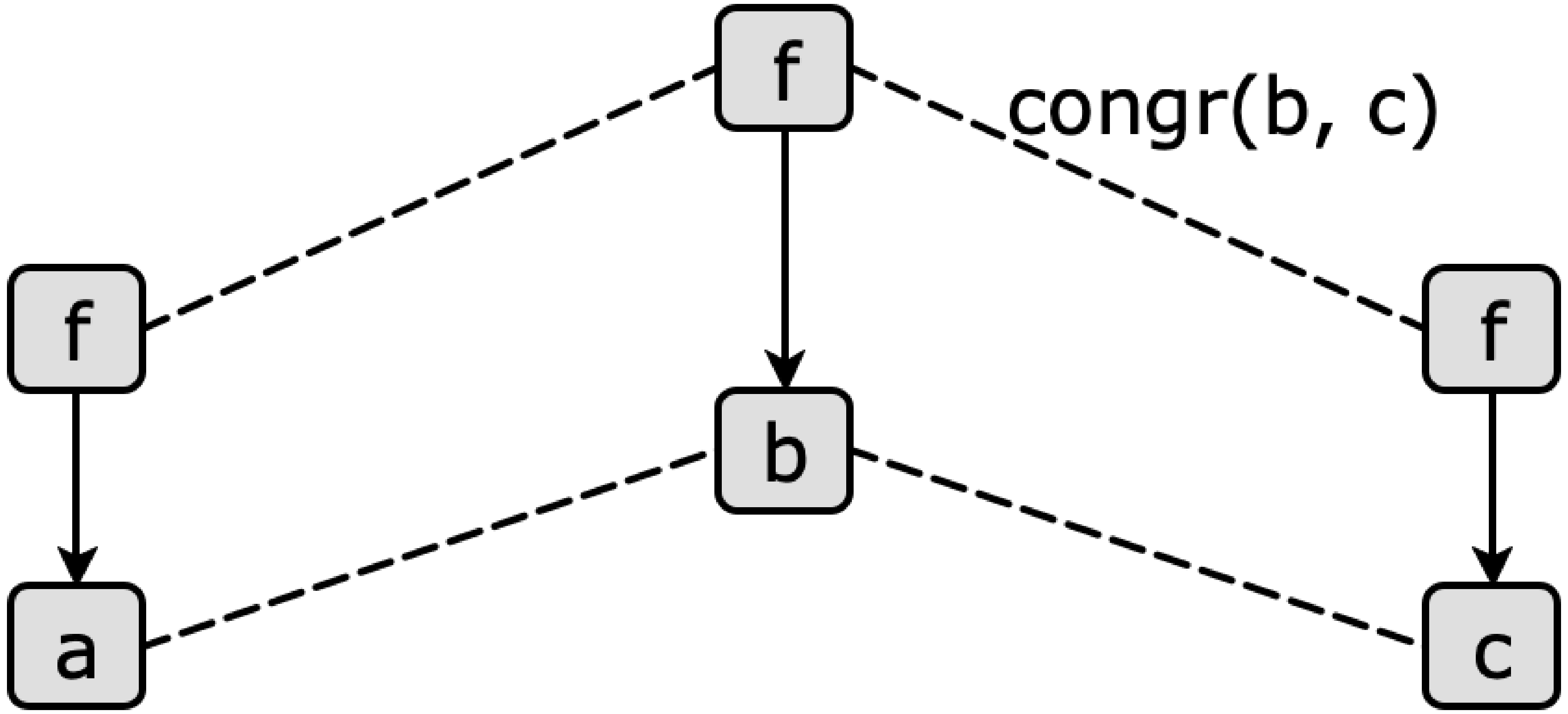


A Bigger E-Graph Example

Inputs:

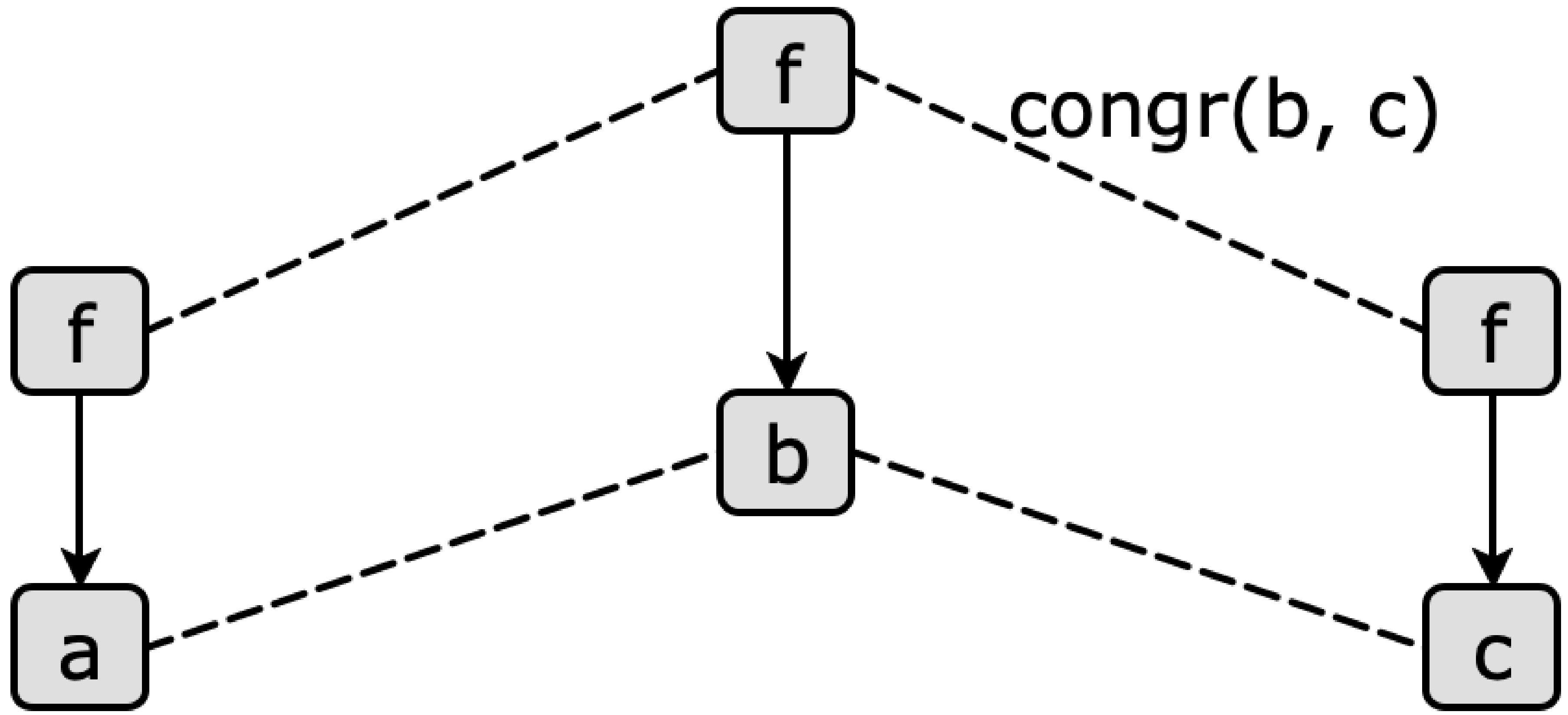
f(c)

f(a) = f(b)

a = b

b = c

a = c unnecessary



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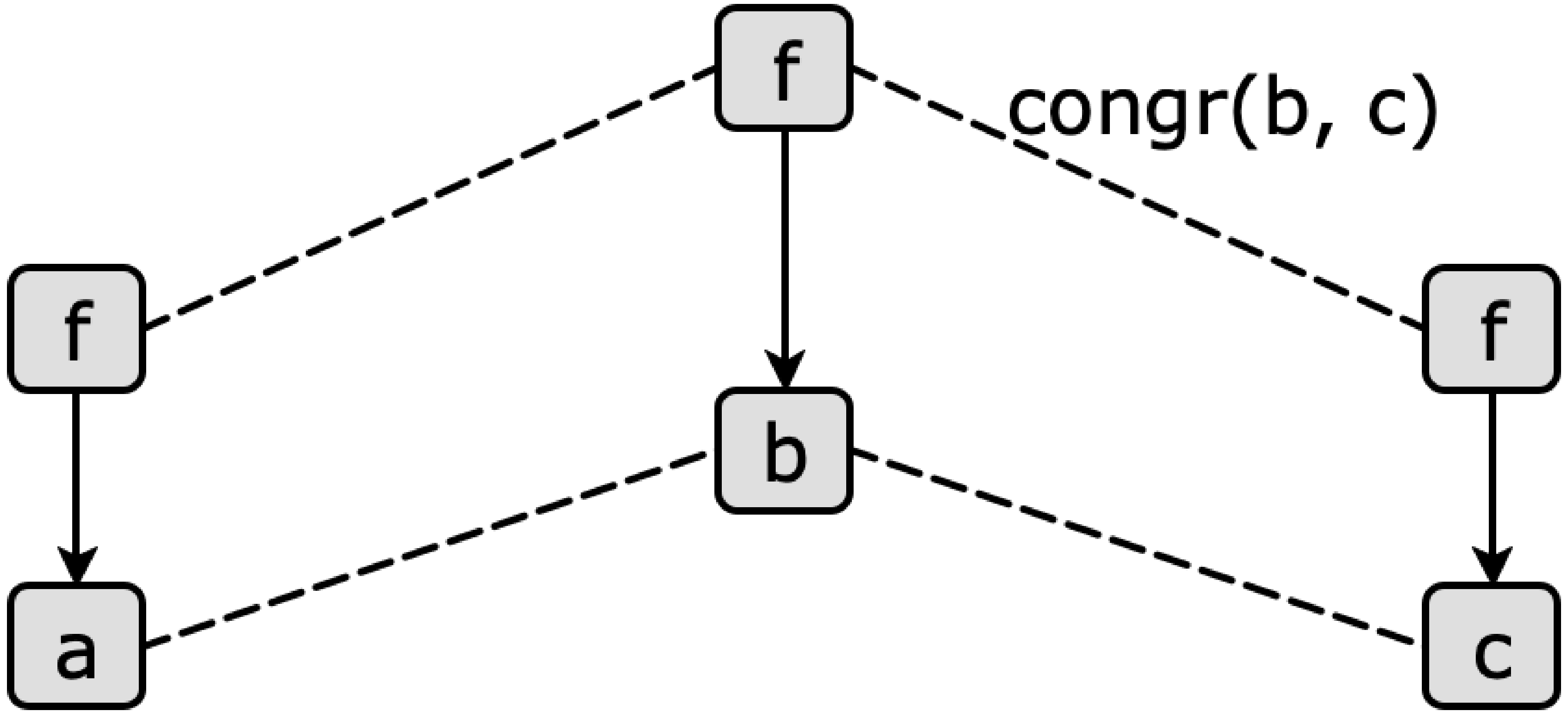

Are a and c equal? Yes!

Are f(a) and a? No!

Are f(a) and f(c)? Yes!

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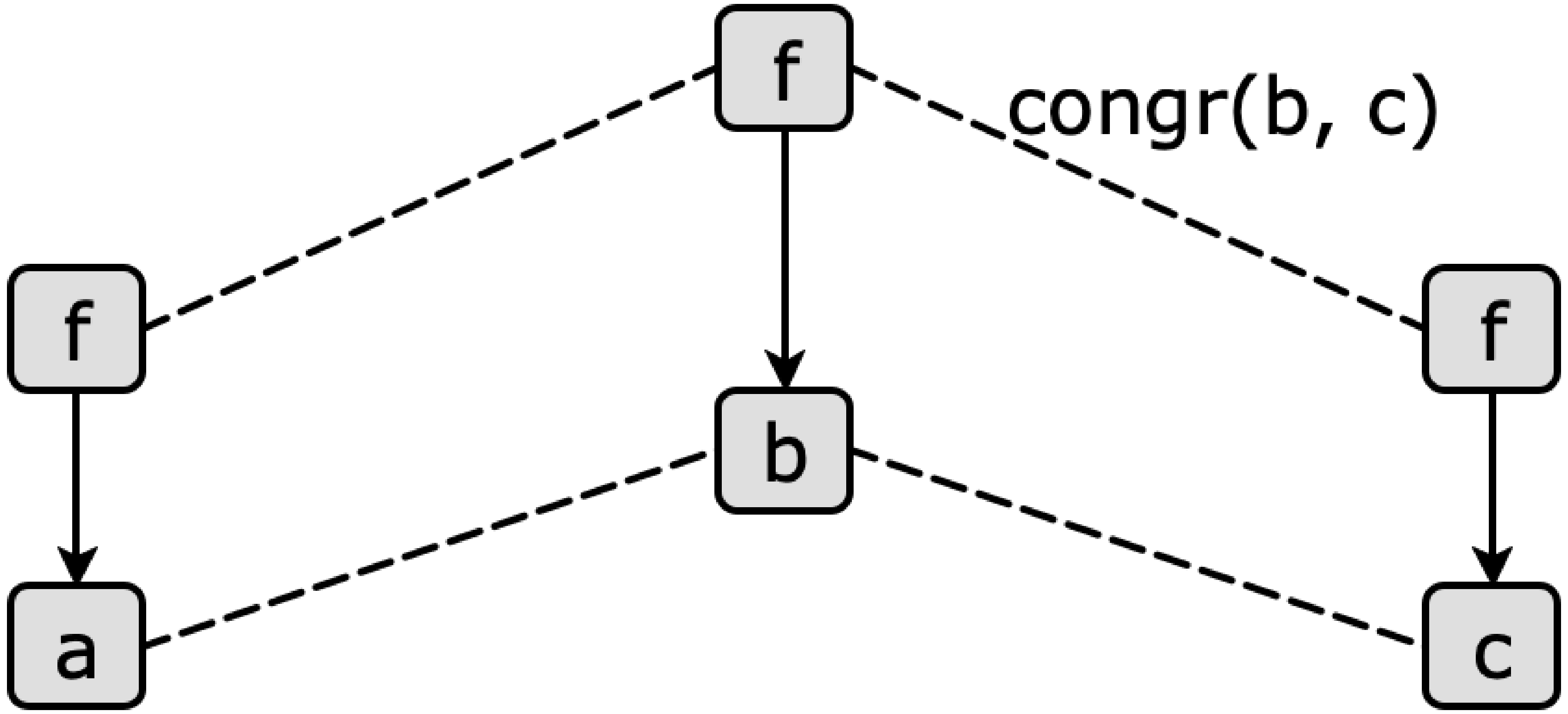

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Key idea: equality edges form equivalence classes of terms

Example:

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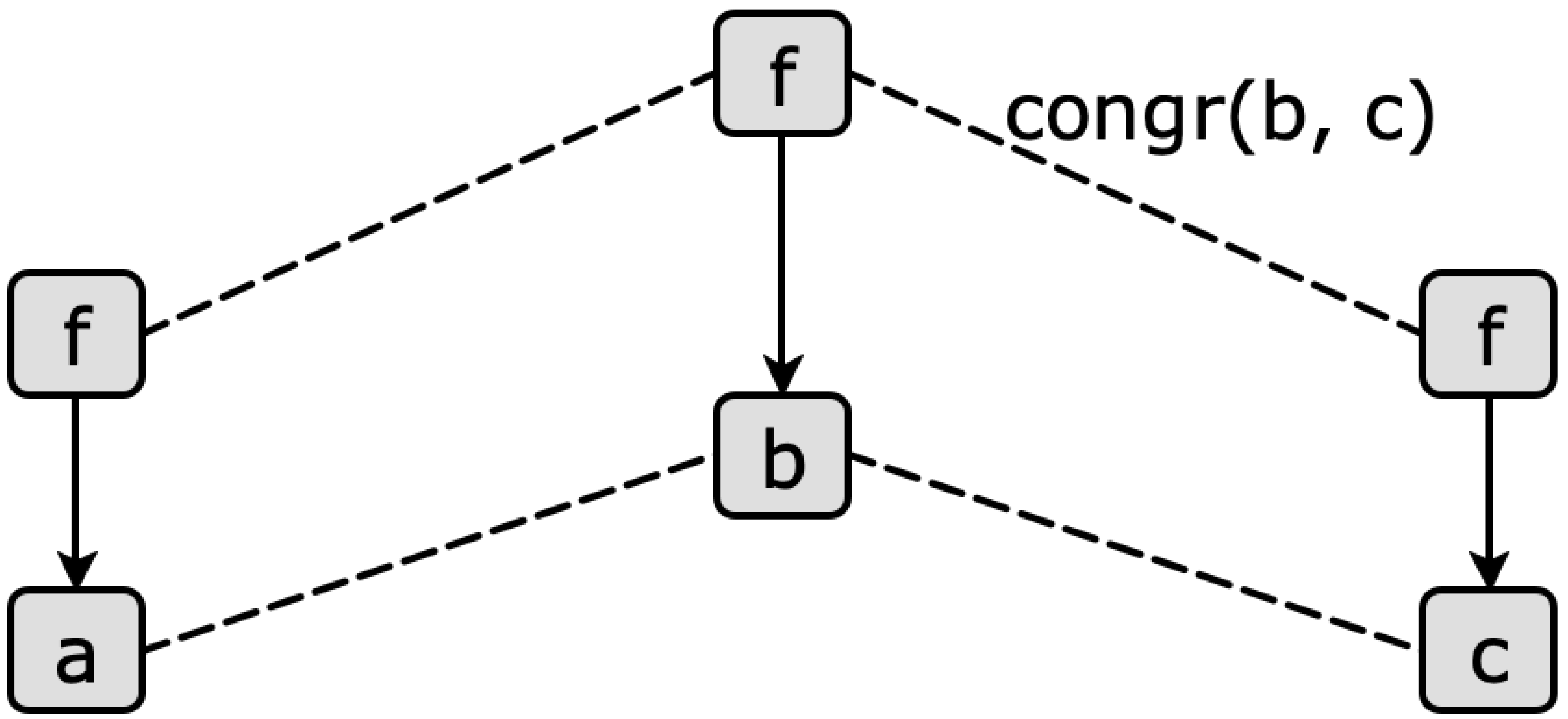

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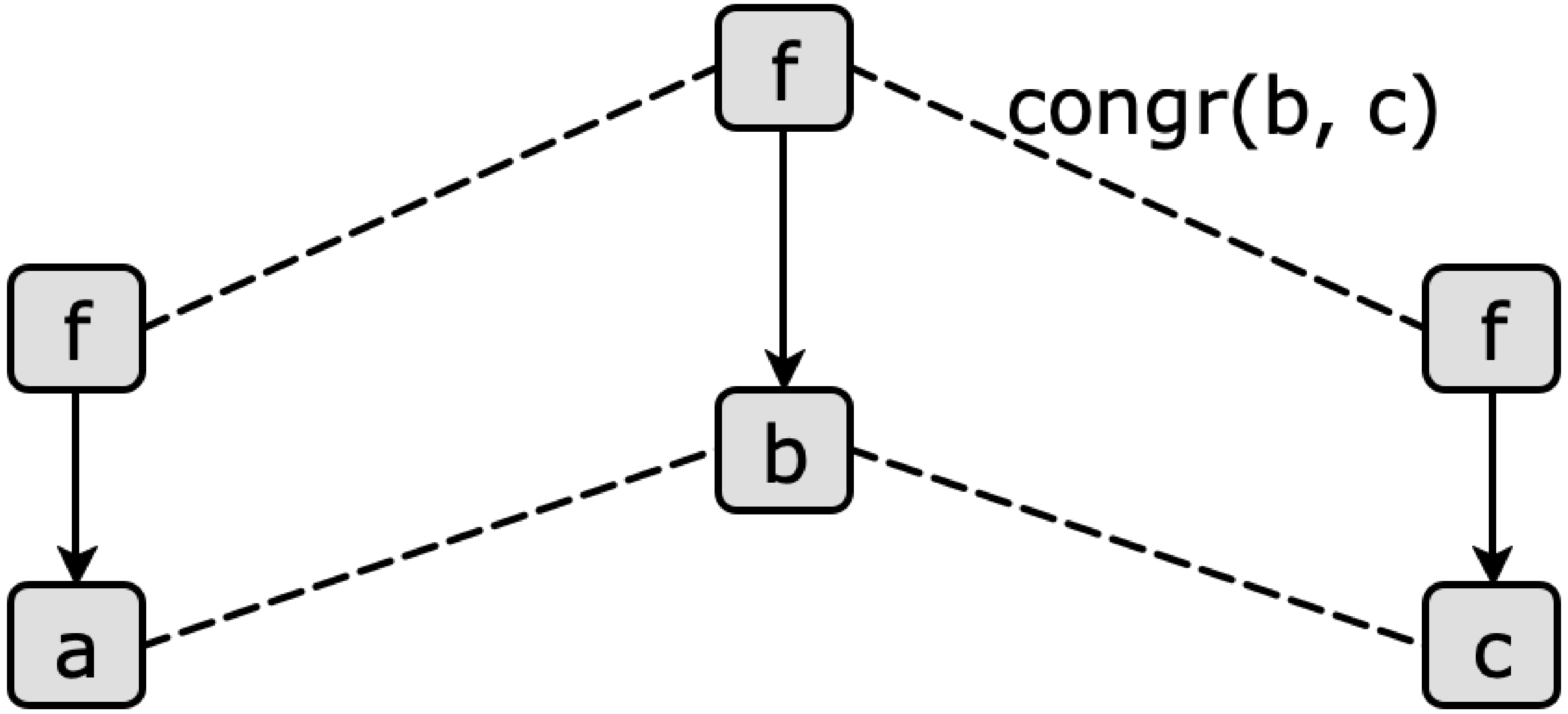
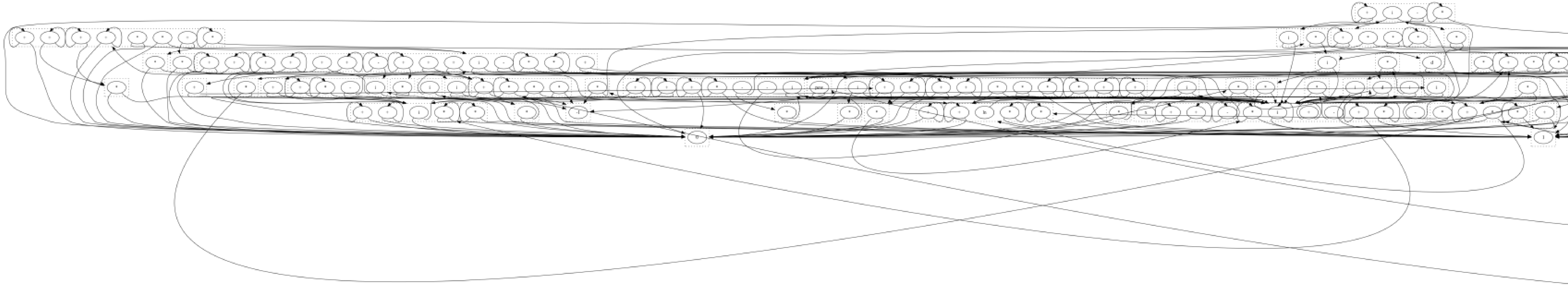


Reality:



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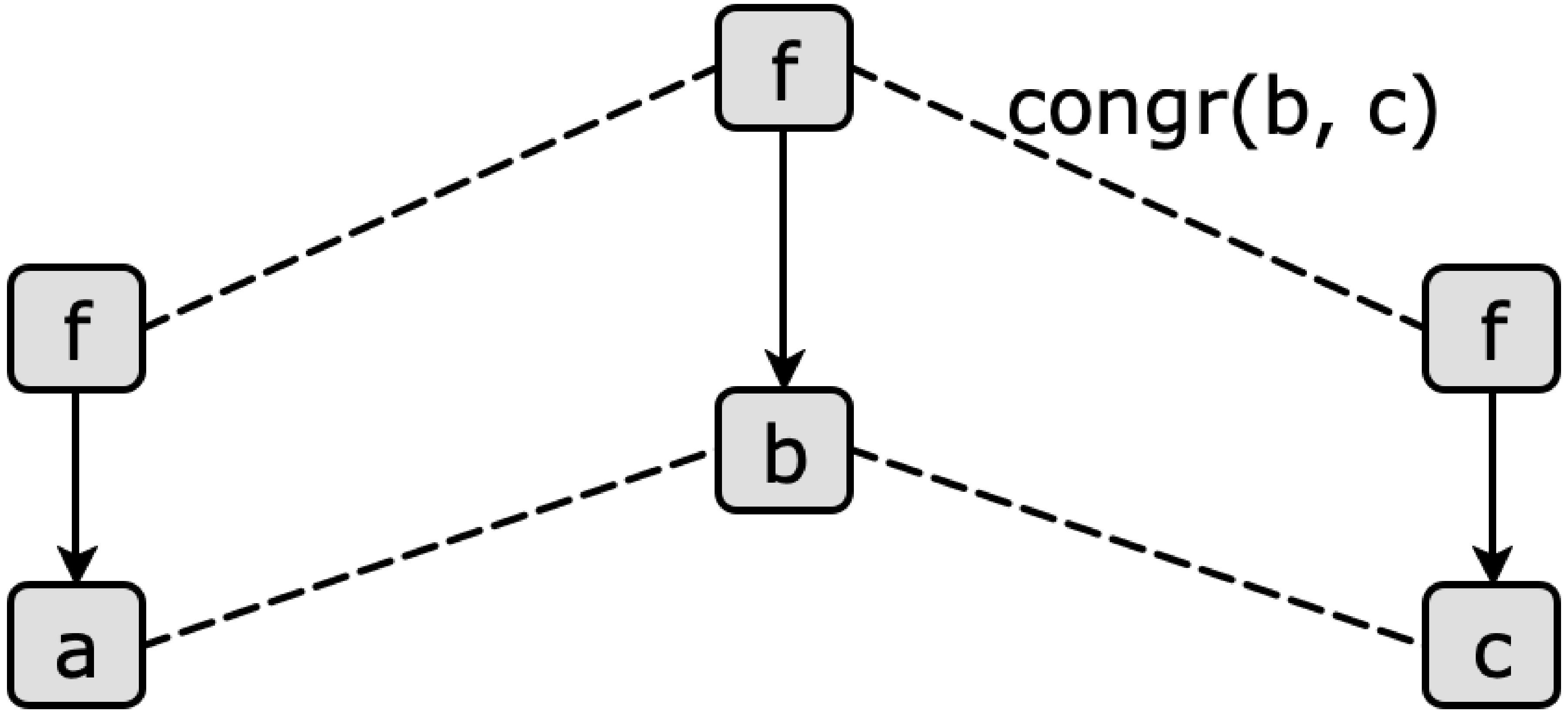

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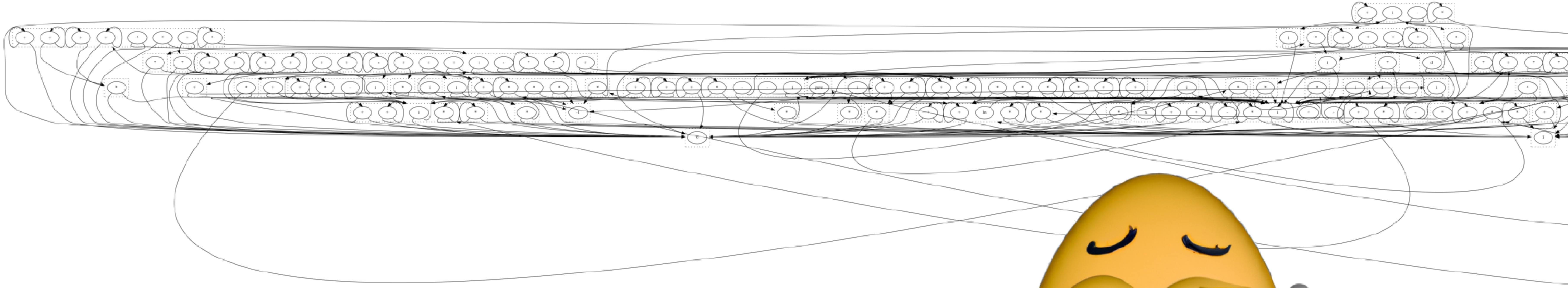

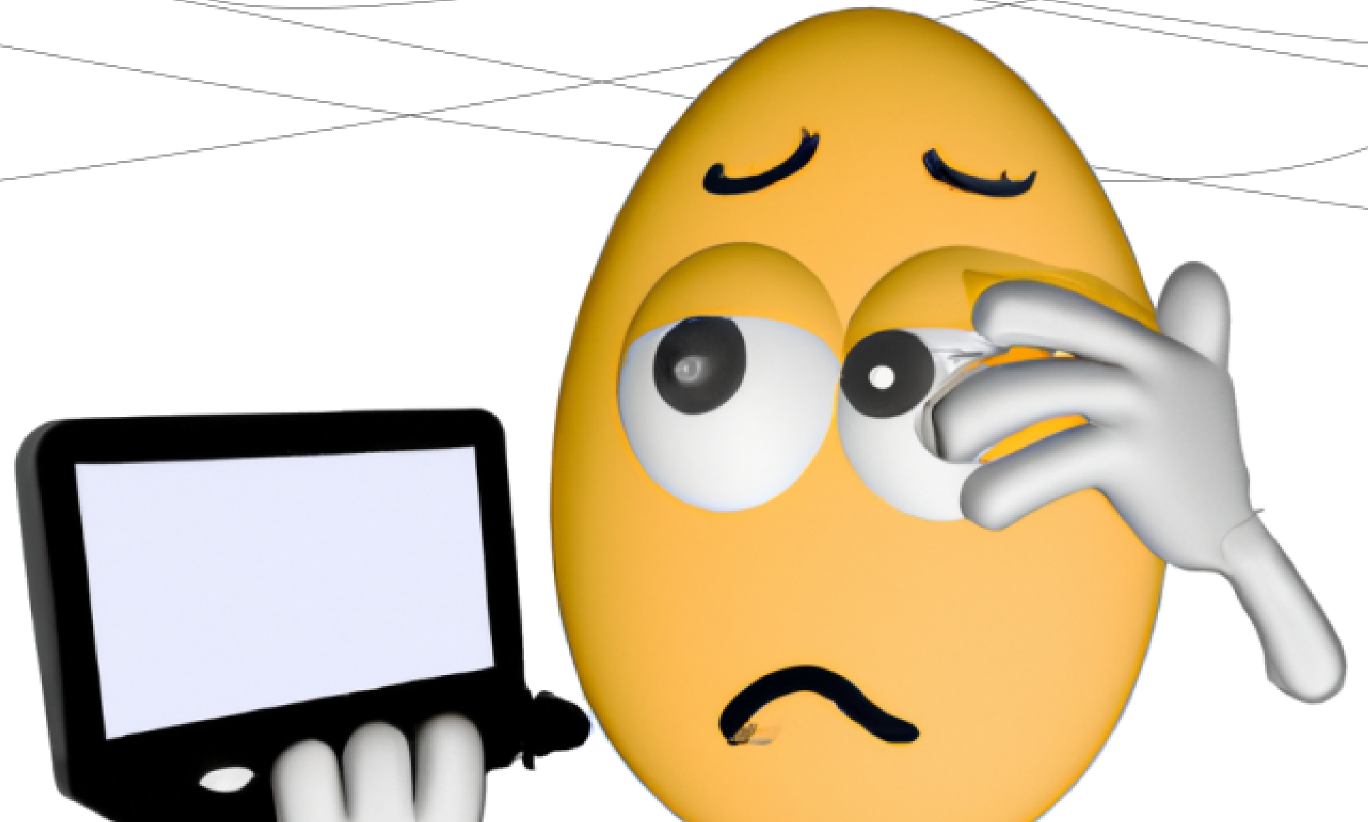

- [x] Motivation
- [x] Congruence Closure
- [ ] Proofs from Congruence Closure
- [ ] Finding Small Proofs

27.2% smaller proofs!

Congruence Proofs

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Answer the question "how are these two terms equal?"

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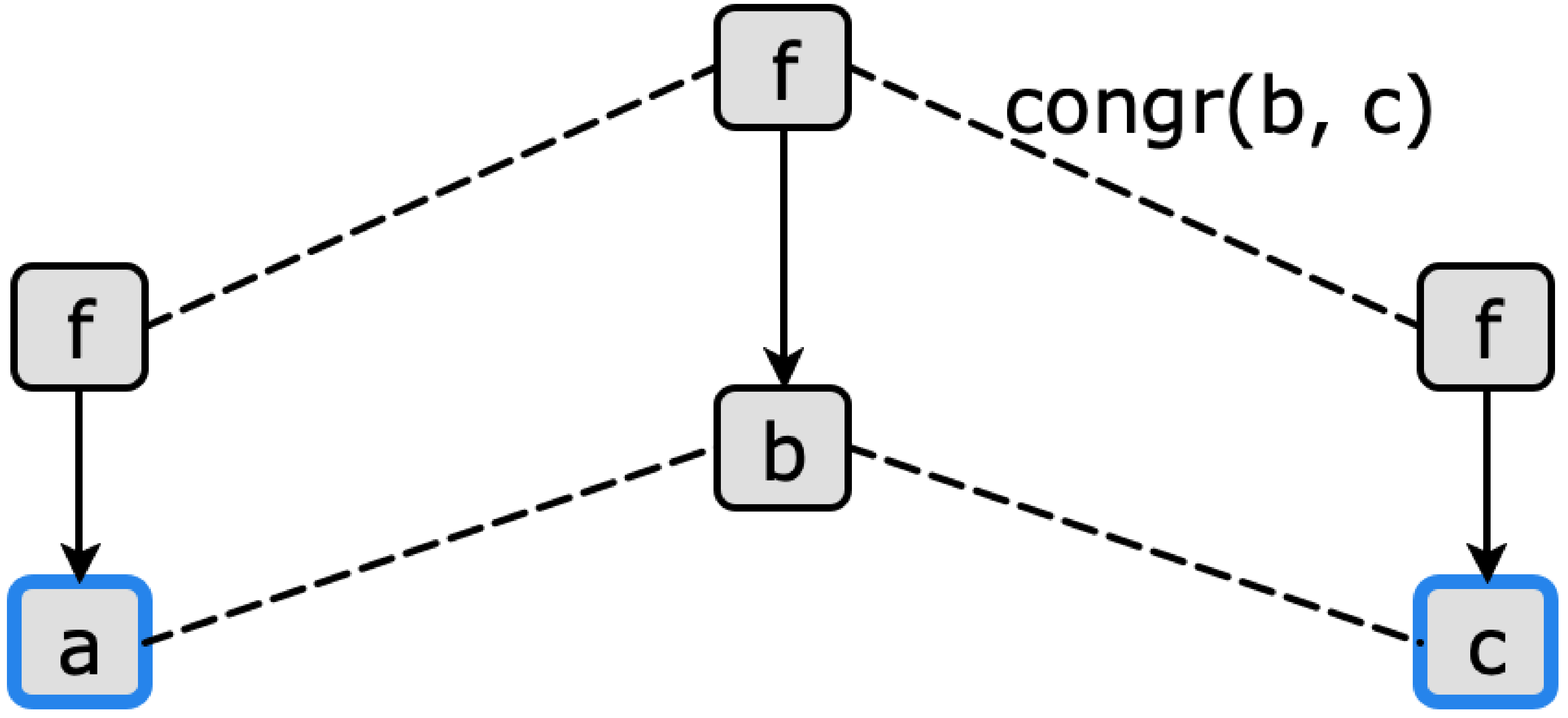

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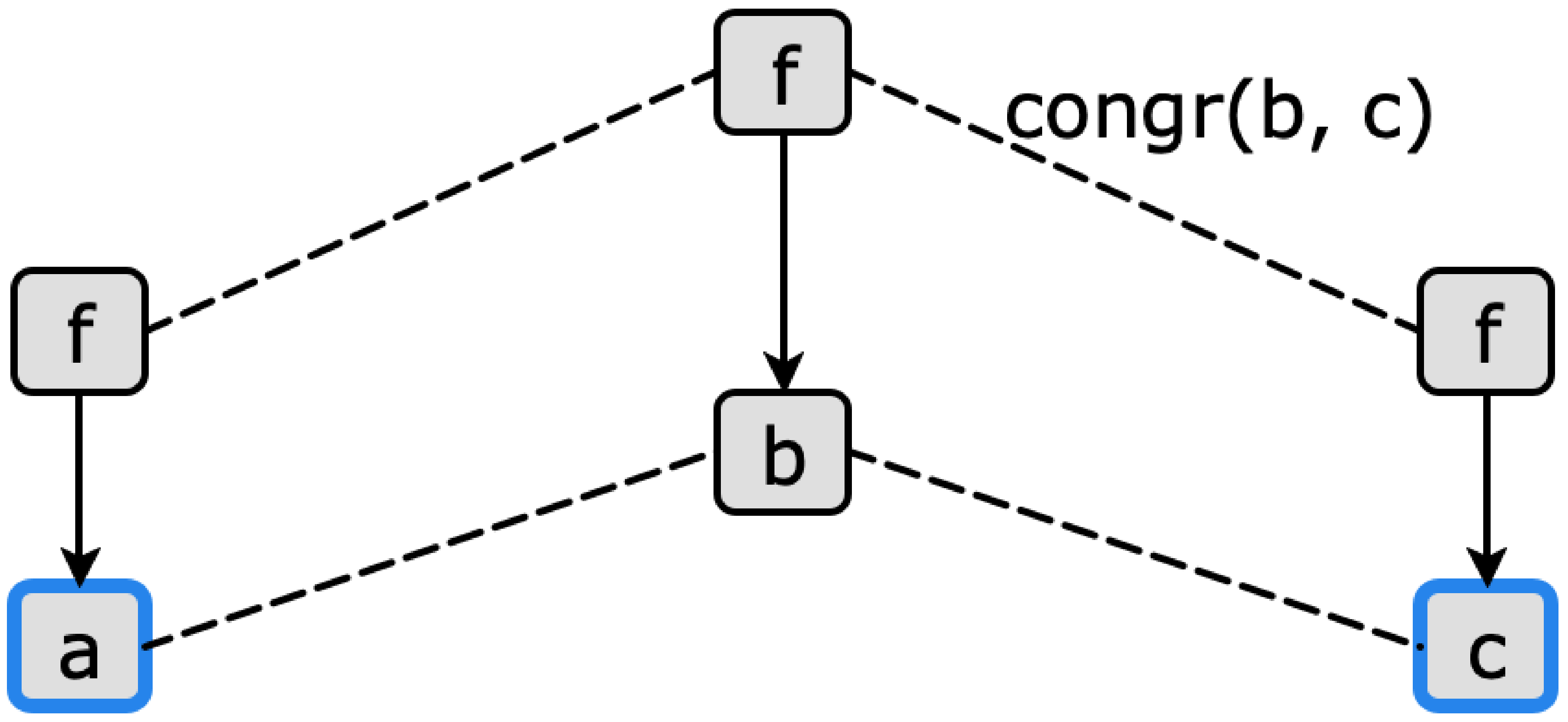

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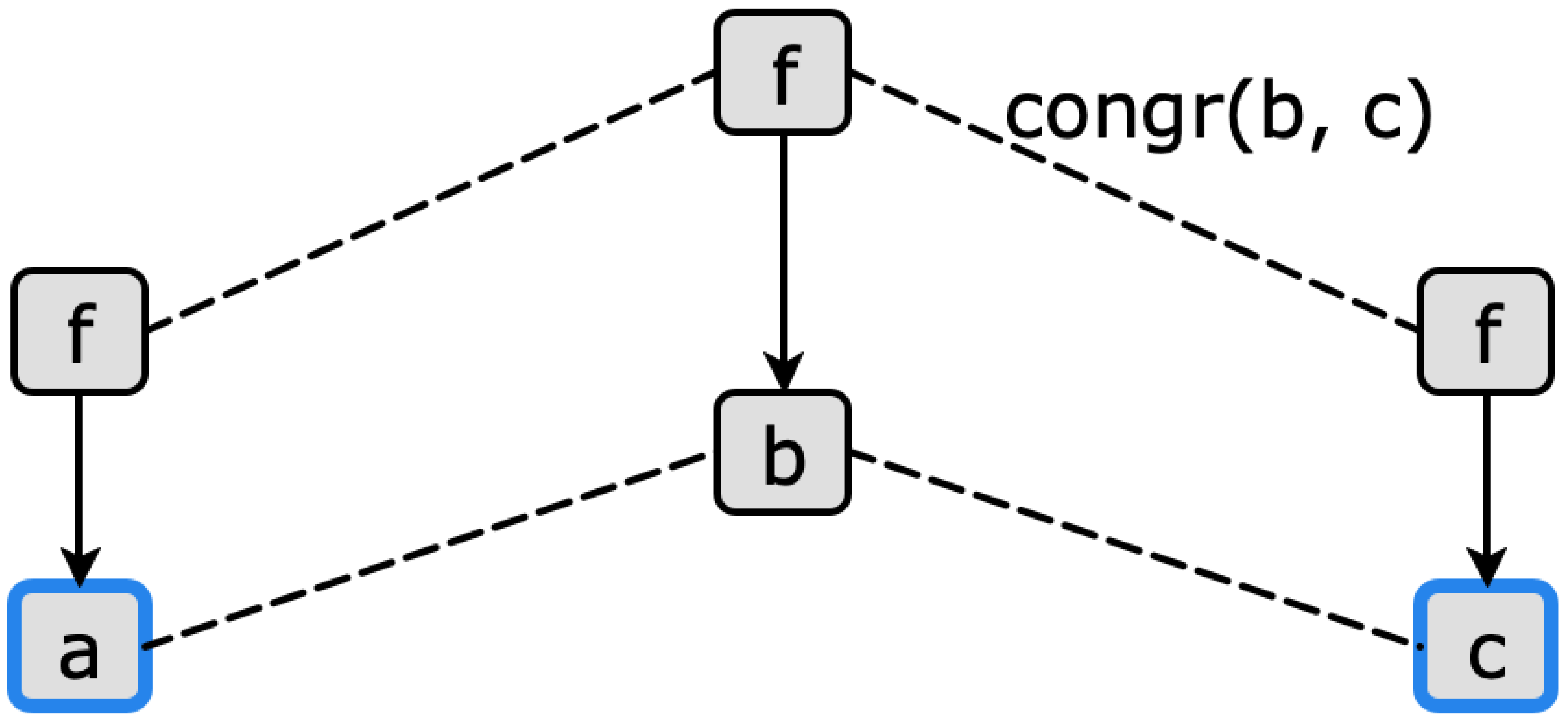

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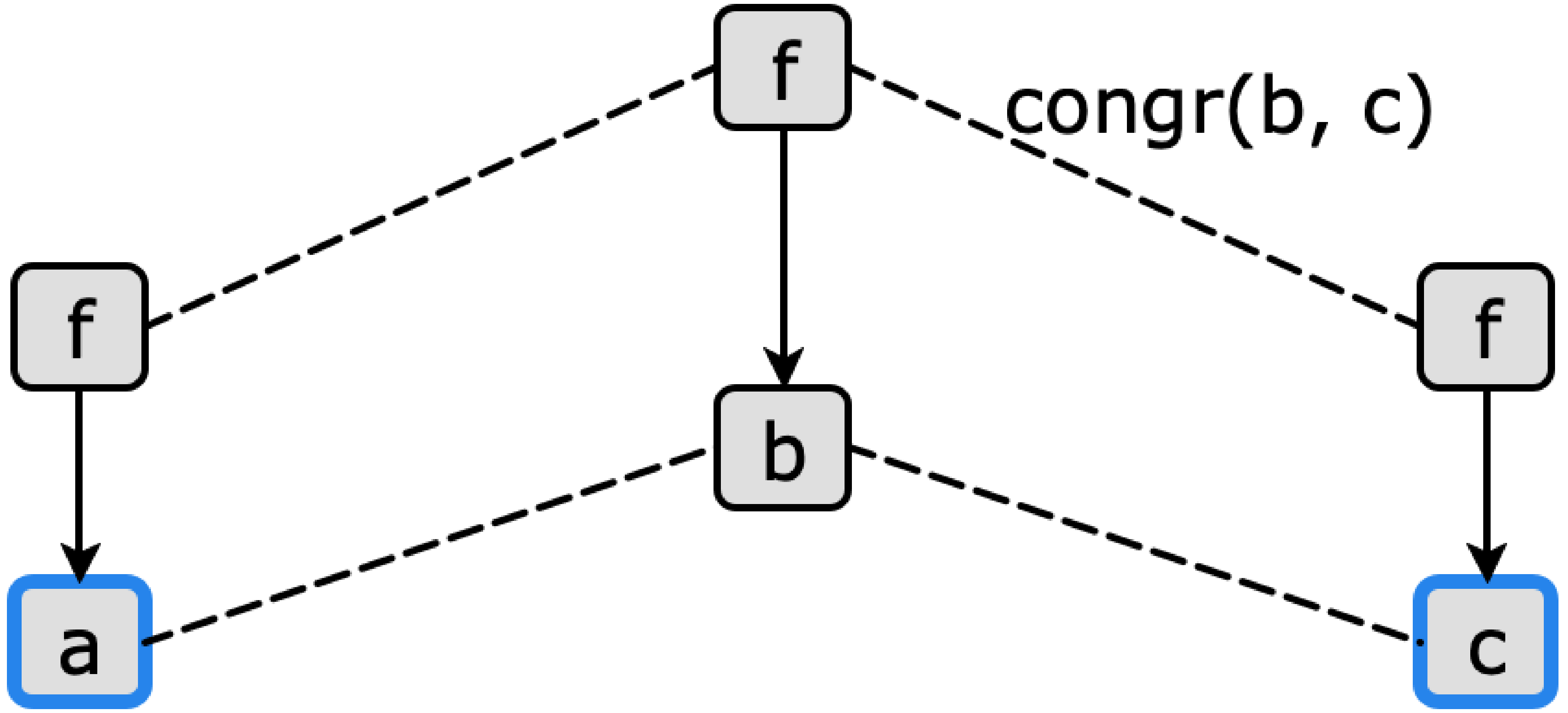

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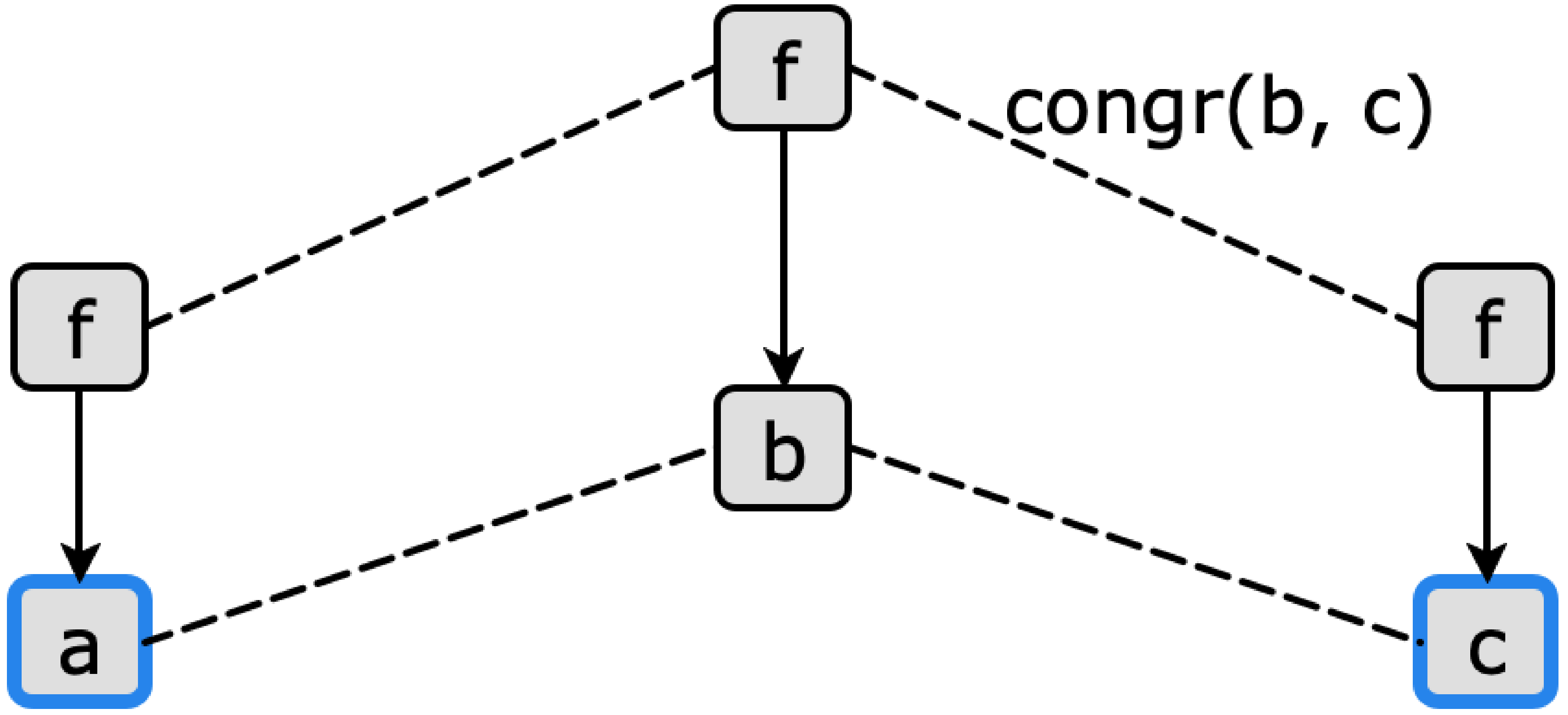

Prove a and c are equal:

a = b

b = c

done!

Inputs:

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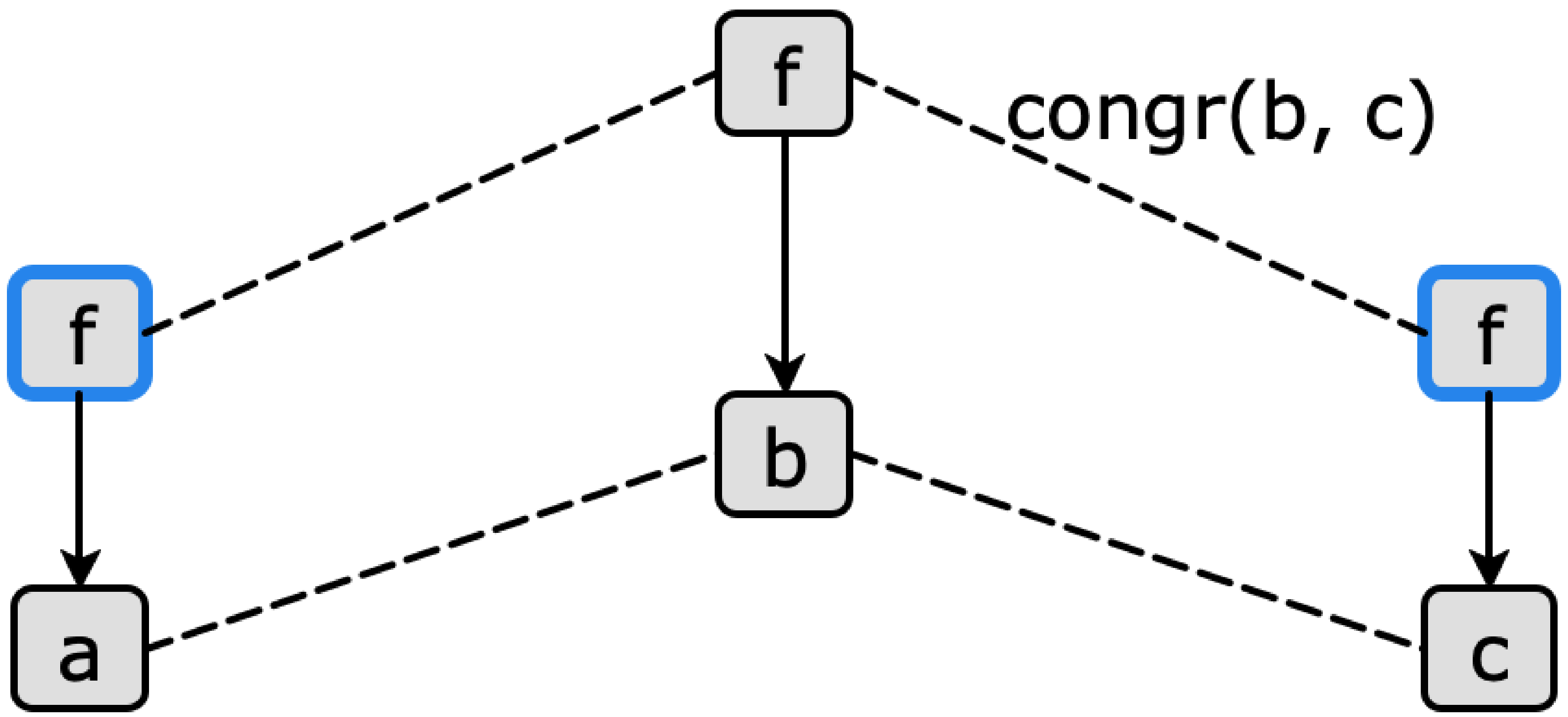

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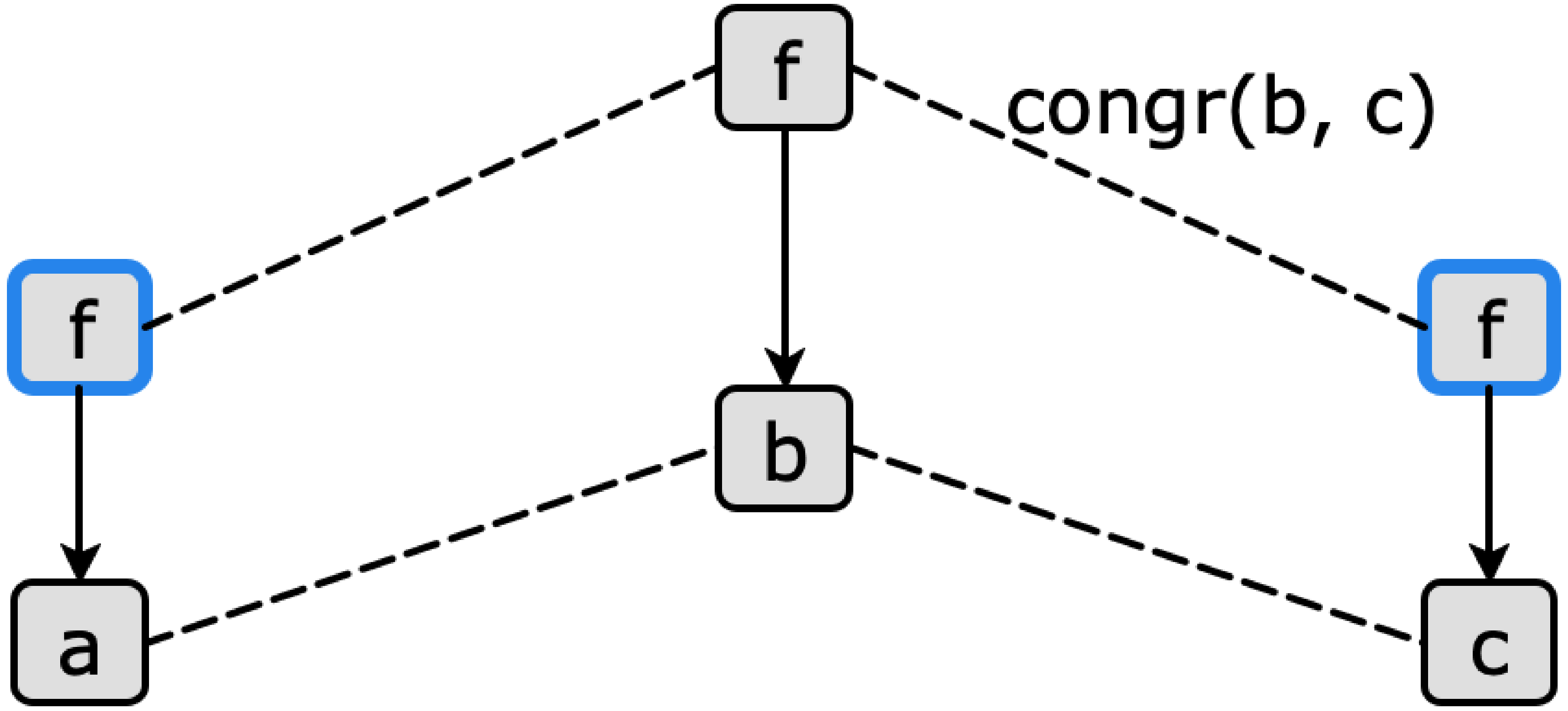

Prove f(a) and f(c) are equal:

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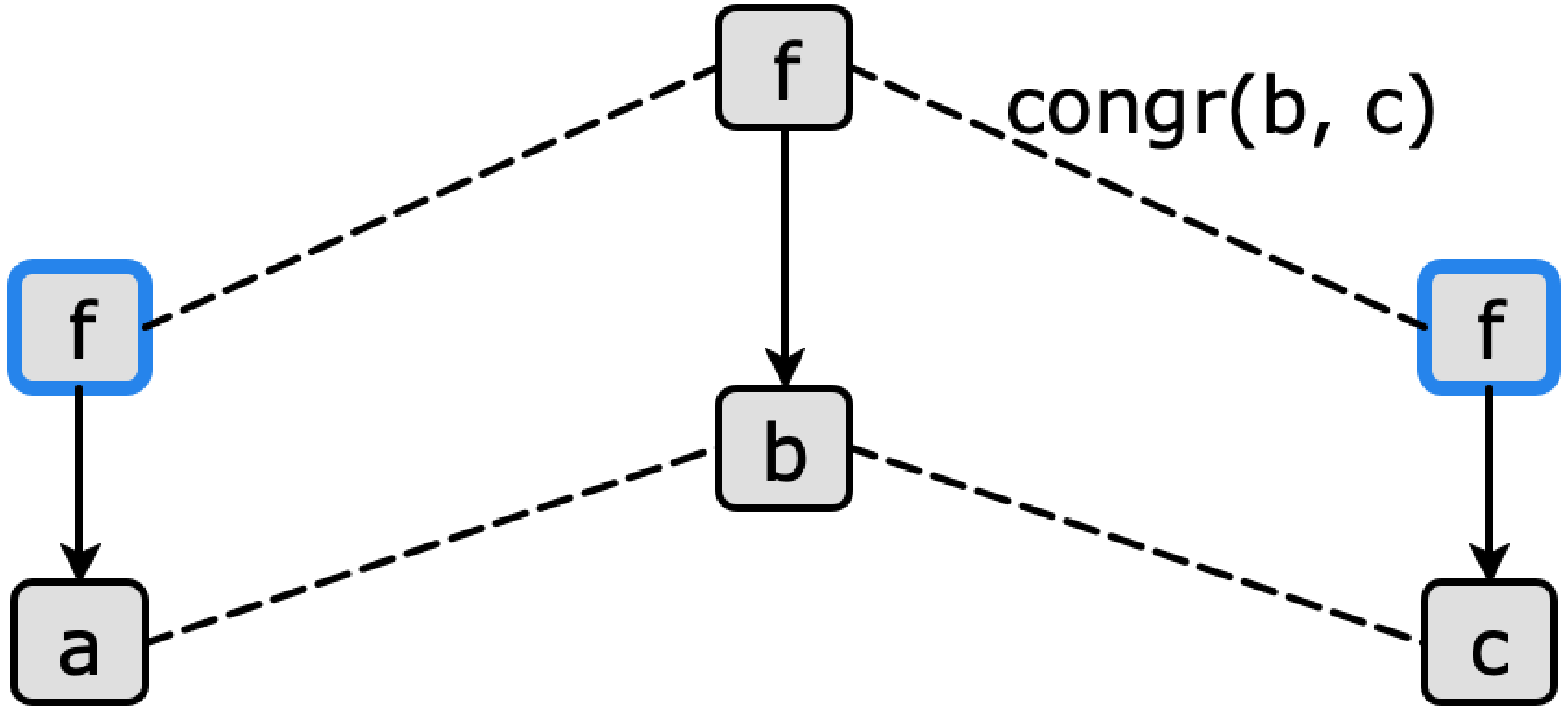

Prove f(a) and f(c) are equal:

f(a) = f(b)

Prove f(b) = f(c) by congruence:

Inputs:

f(c)

f(a) = f(b)

a = b

b = c

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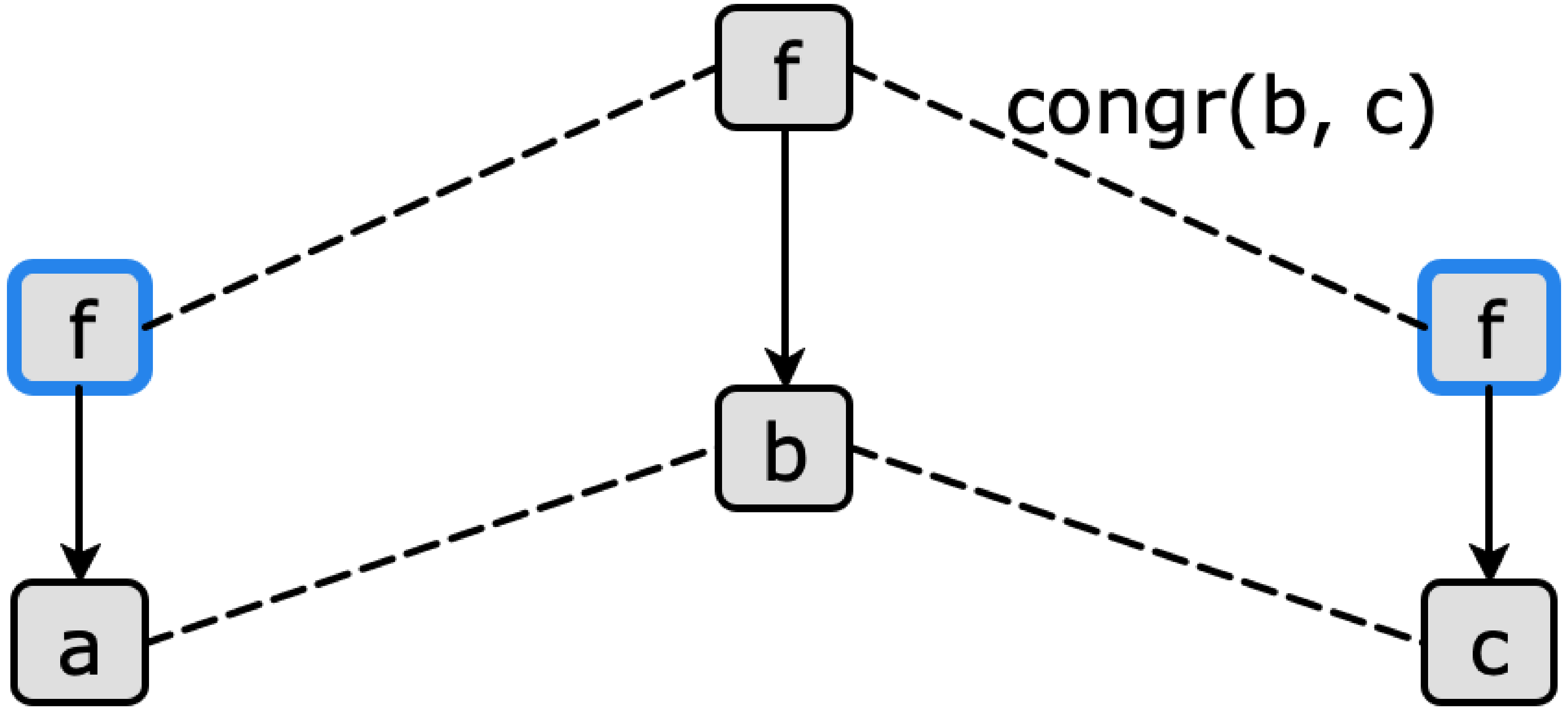

Prove f(a) and f(c) are equal:

- f(a) = f(b)
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  - b = c

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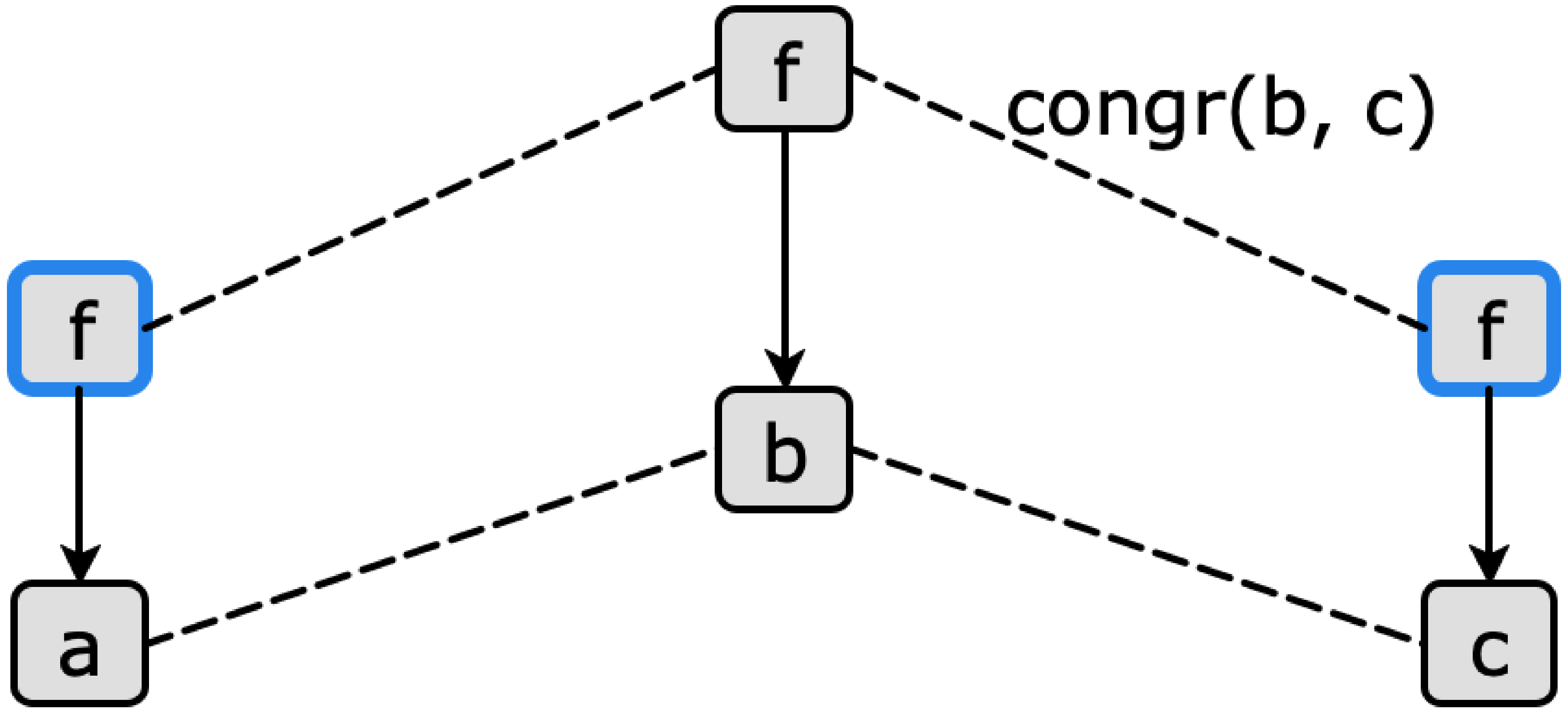

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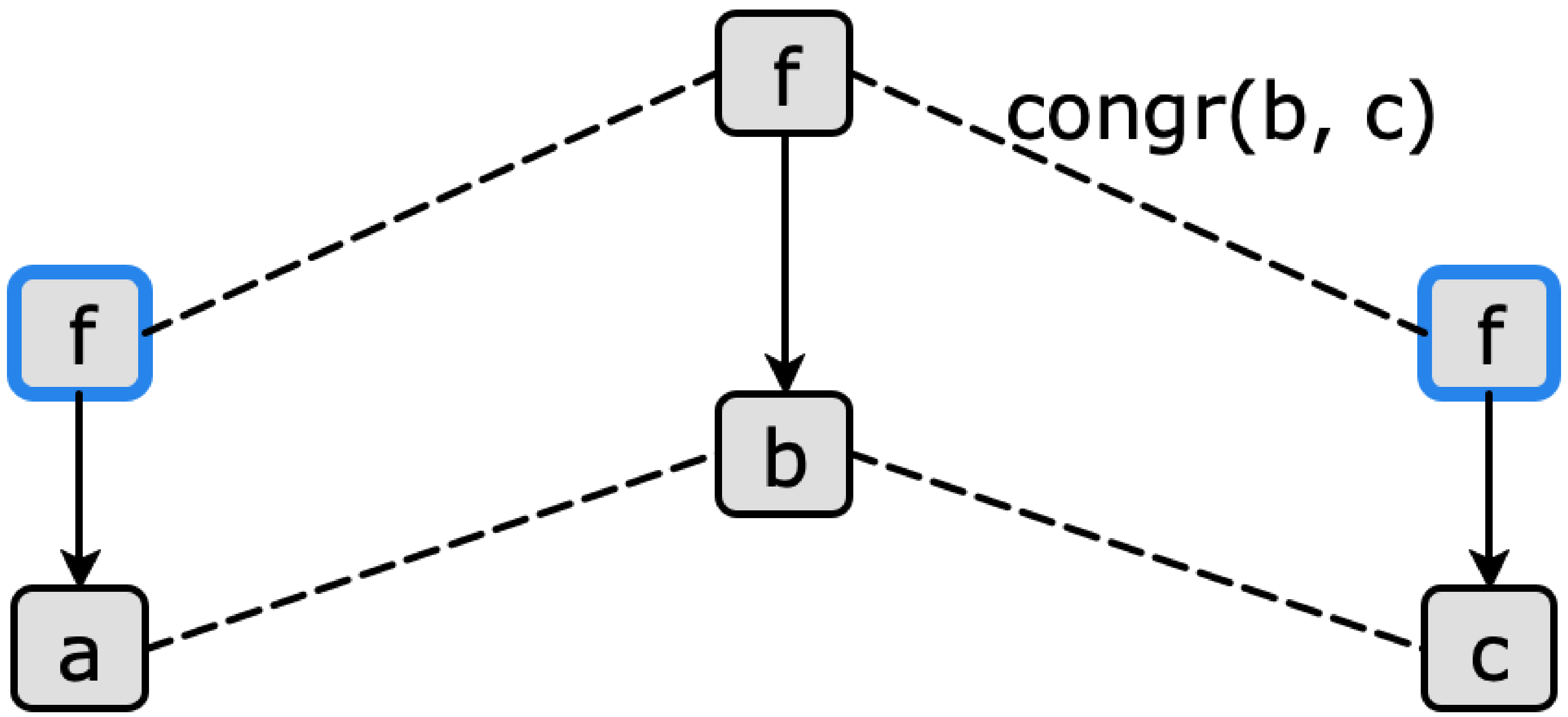

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done!

Prove f(a) and f(c) are equal:

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- done!

We define **proof size** as the number of **unique** equalities in the proof

Prove f(a) and f(c) are equal:

f(a) = f(b)

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We define **proof size** as the number of **unique** equalities in the proof

This proof: size 2

Prove f(a) and f(c) are equal:

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Prove f(b) = f(c) by congruence:

b = c

done!

We define **proof size** as the number of **unique** equalities in the proof

This proof: size 2

We can do better!

Prove f(a) and f(c) are equal:

f(a) = f(b)

Prove f(b) = f(c) by congruence:

b = c

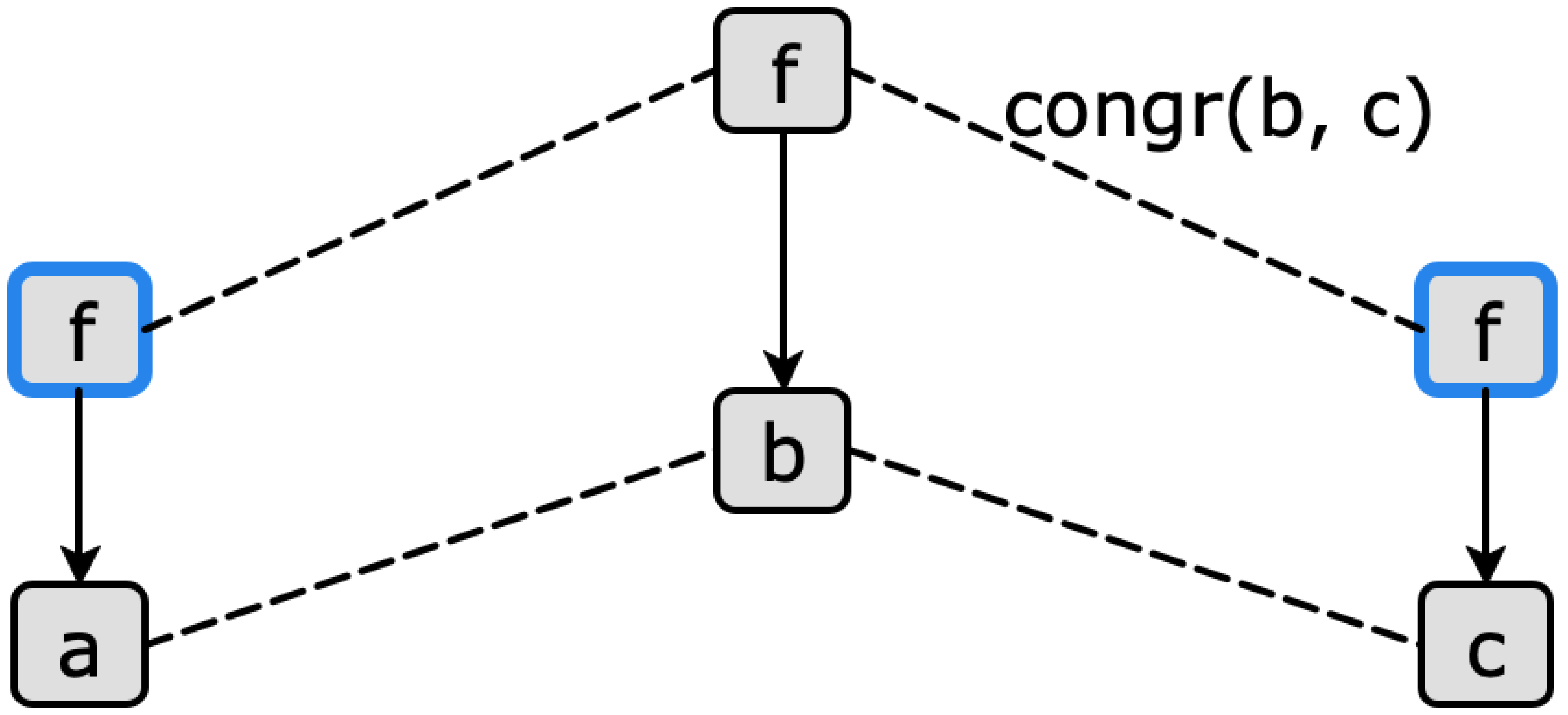
done!

Leveraging Additional Equalities

Inputs:

- f(c)
- f(a) = f(b)
- a = b
- b = c
- a = c

unnecessary



Leveraging Additional Equalities

Inputs:

f(c)

f(a) = f(b)

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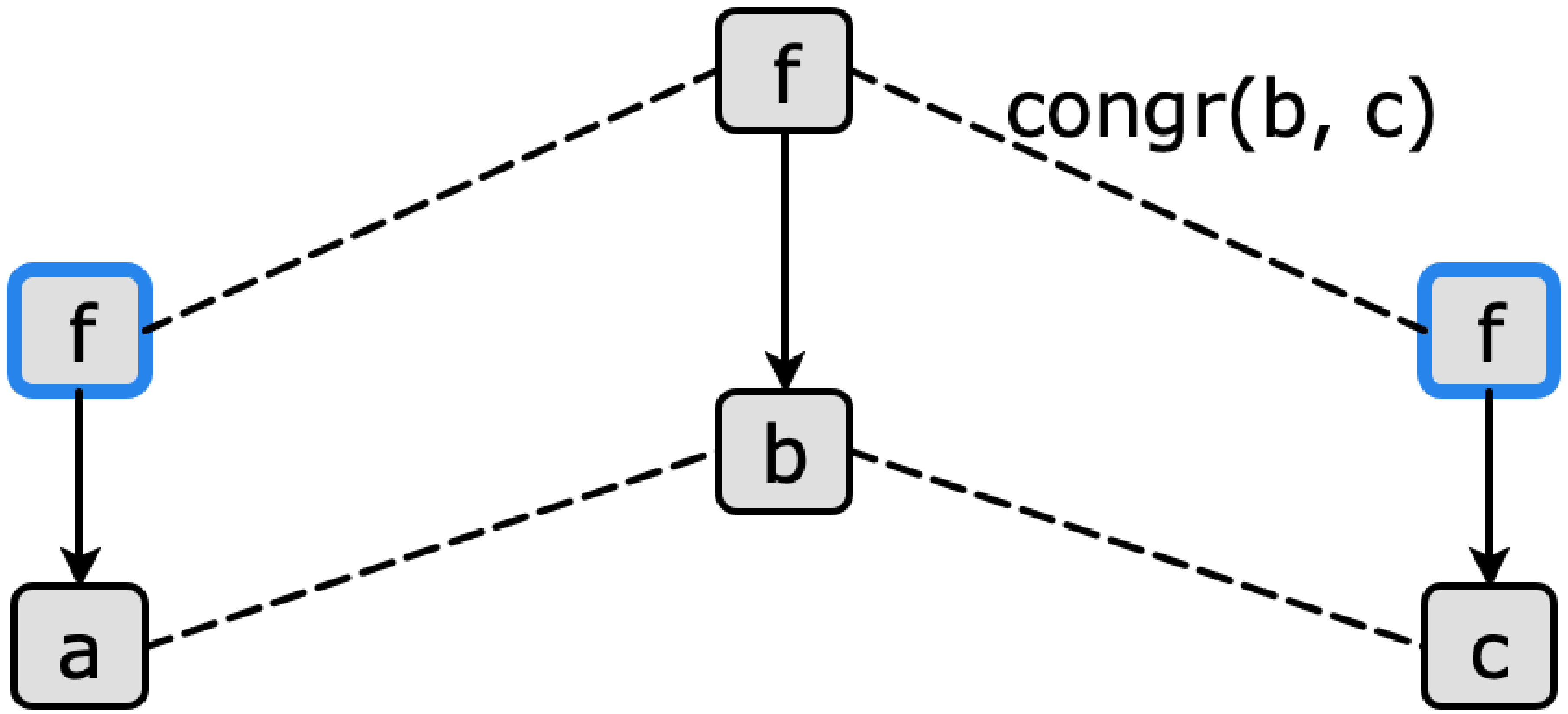
a = c useful



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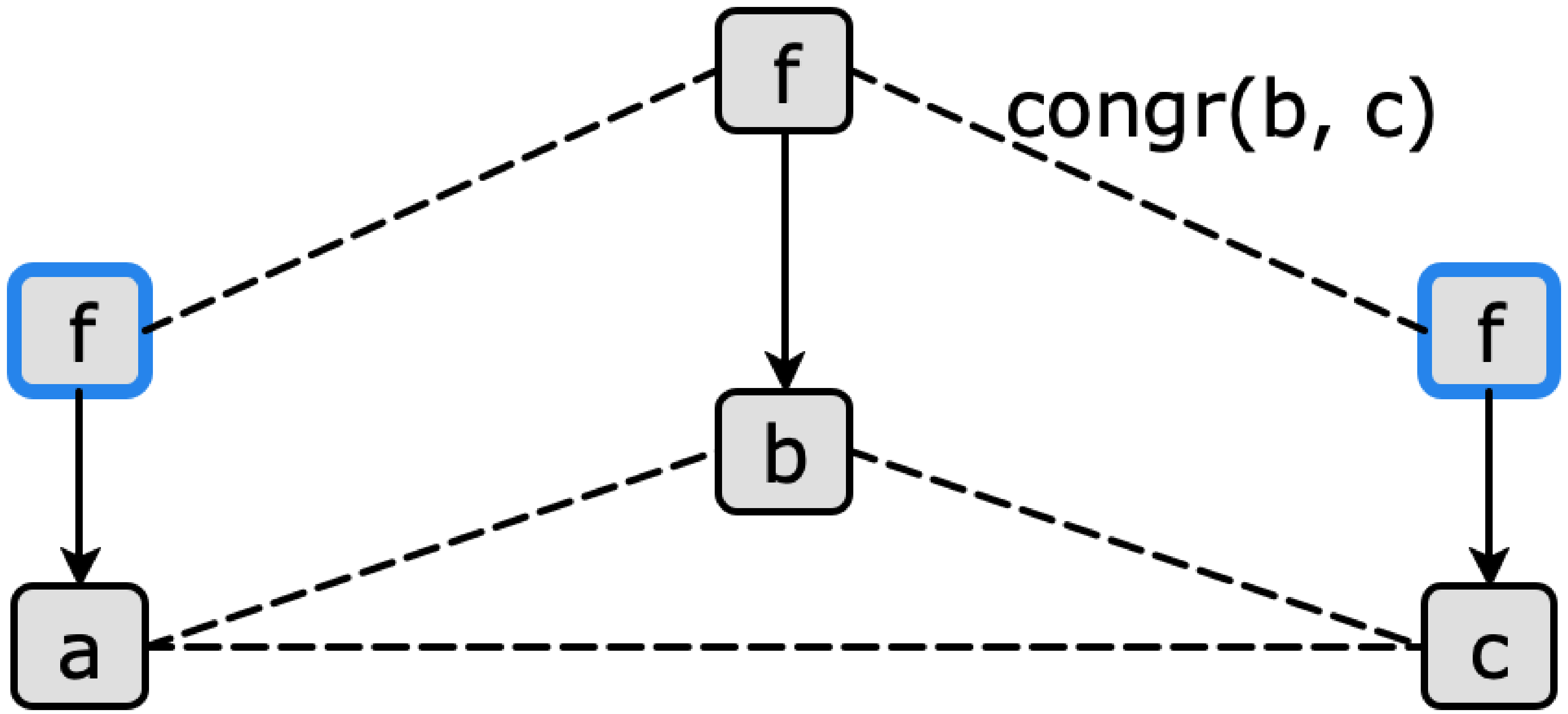

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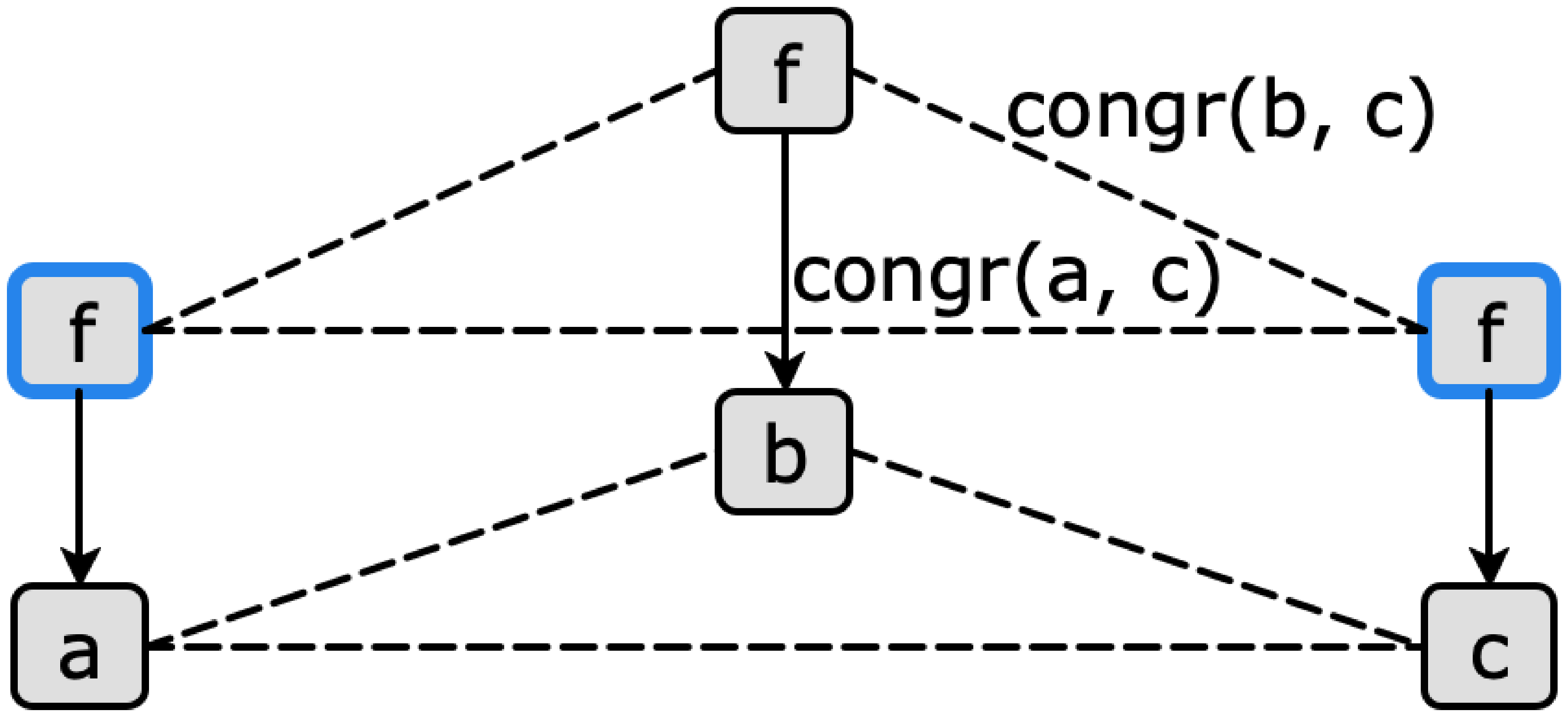

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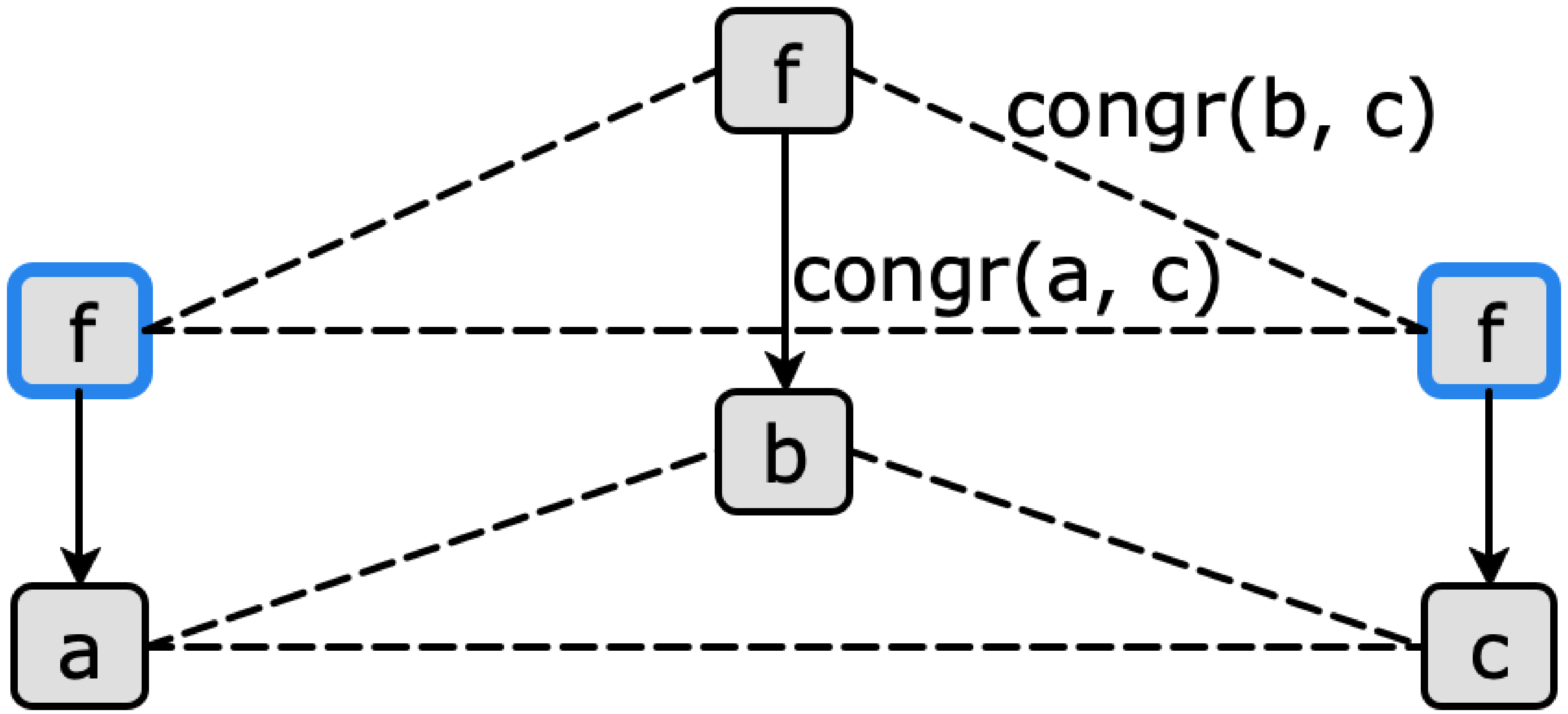
a = c useful

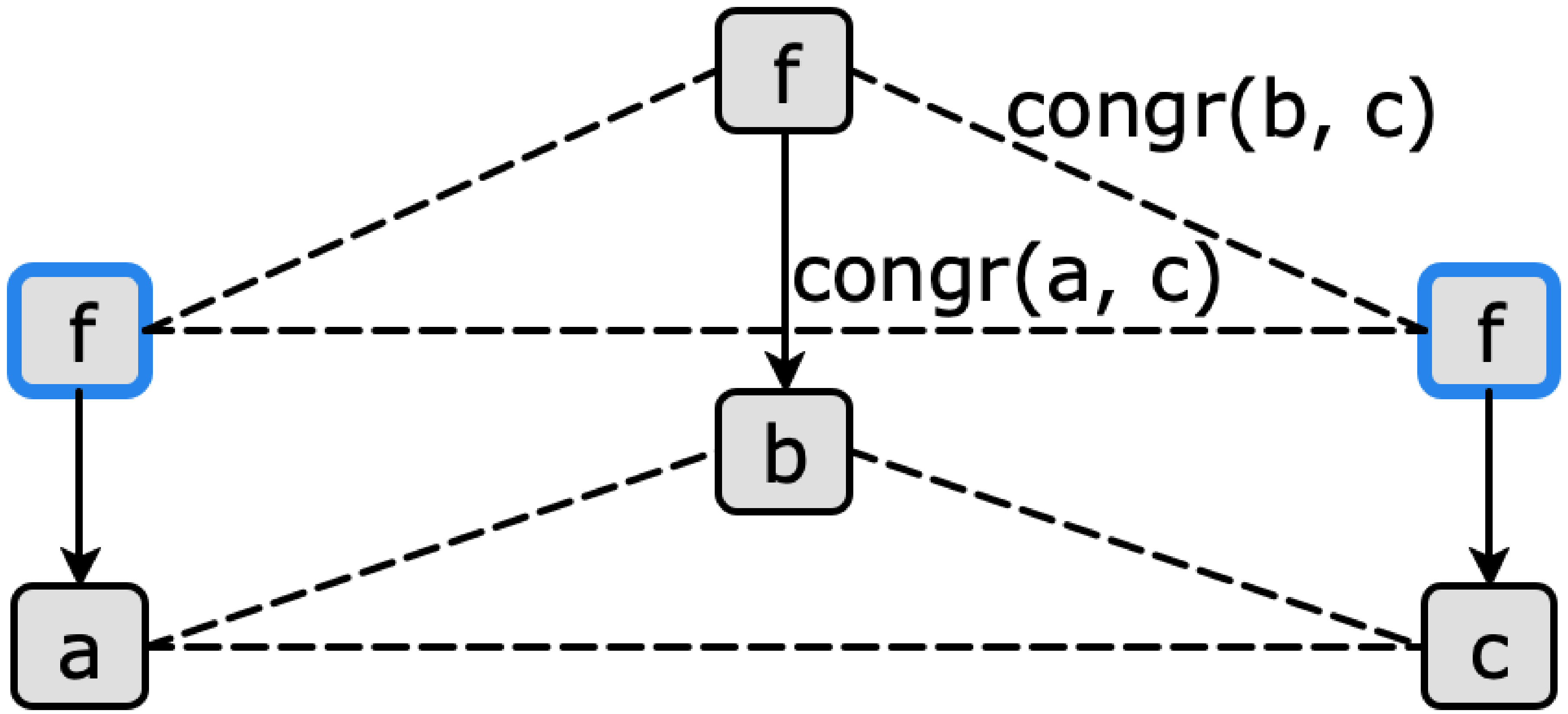


Prove f(a) and f(c) are equal:

Leveraging Additional Equalities

Inputs:

- f(c)
- f(a) = f(b)
- a = b
- b = c
- a = c useful



Prove f(a) and f(c) are equal:

Prove f(a) = f(c) by congruence:

Leveraging Additional Equalities

Inputs:

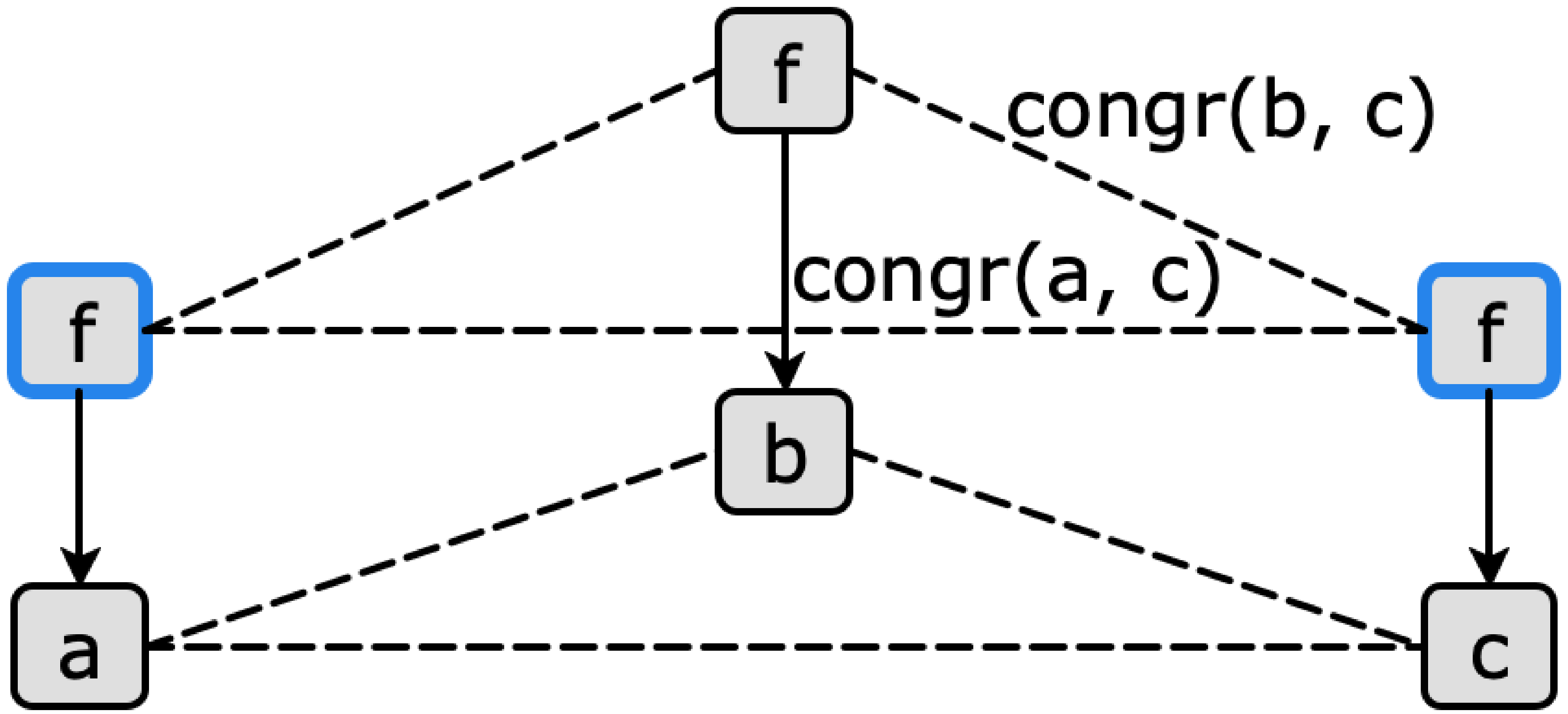
f(c)

f(a) = f(b)

a = b

b = c

a = c useful



Prove f(a) and f(c) are equal:

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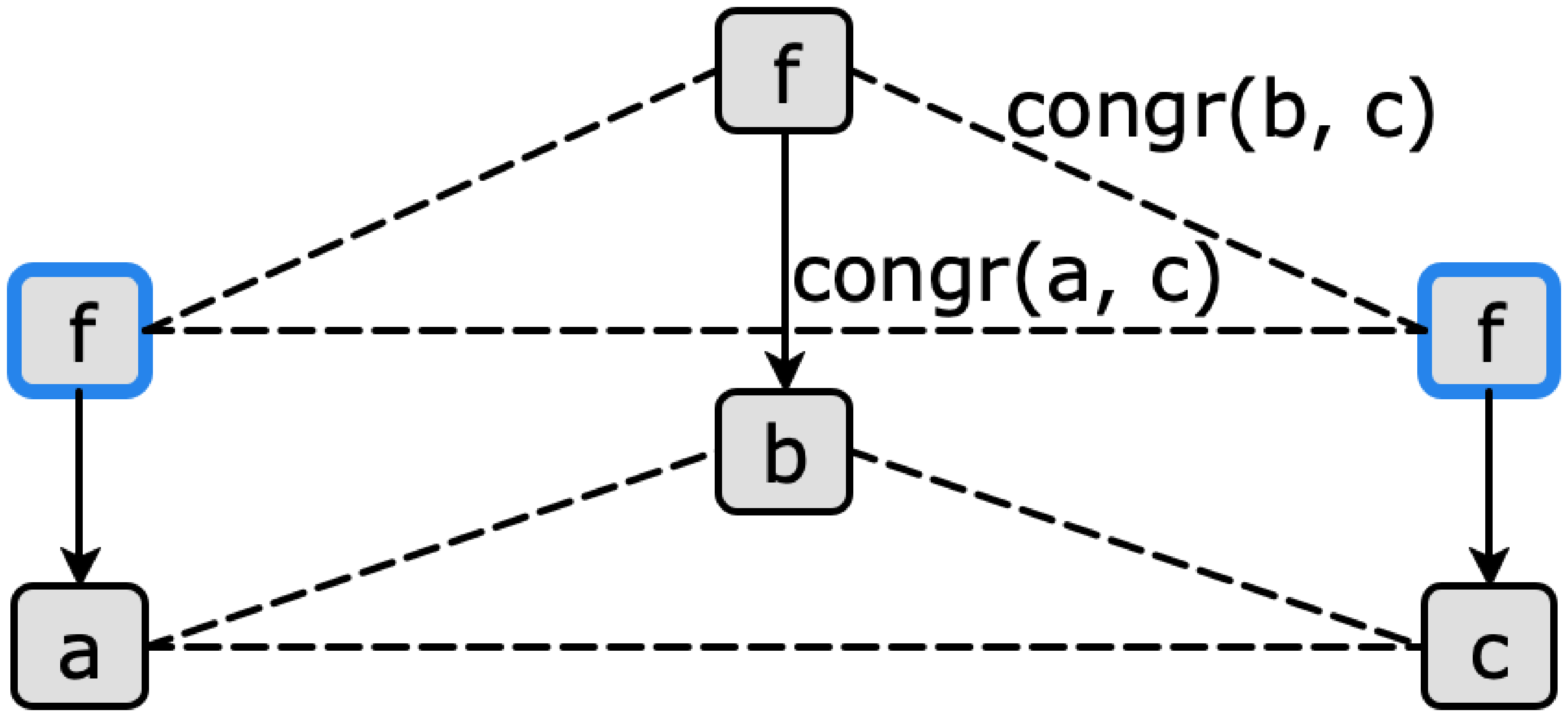

Prove f(a) and f(c) are equal:

Prove f(a) = f(c) by congruence:

a = c

done!

Leveraging Additional Equalities

Inputs:

f(c)

f(a) = f(b)

a = b

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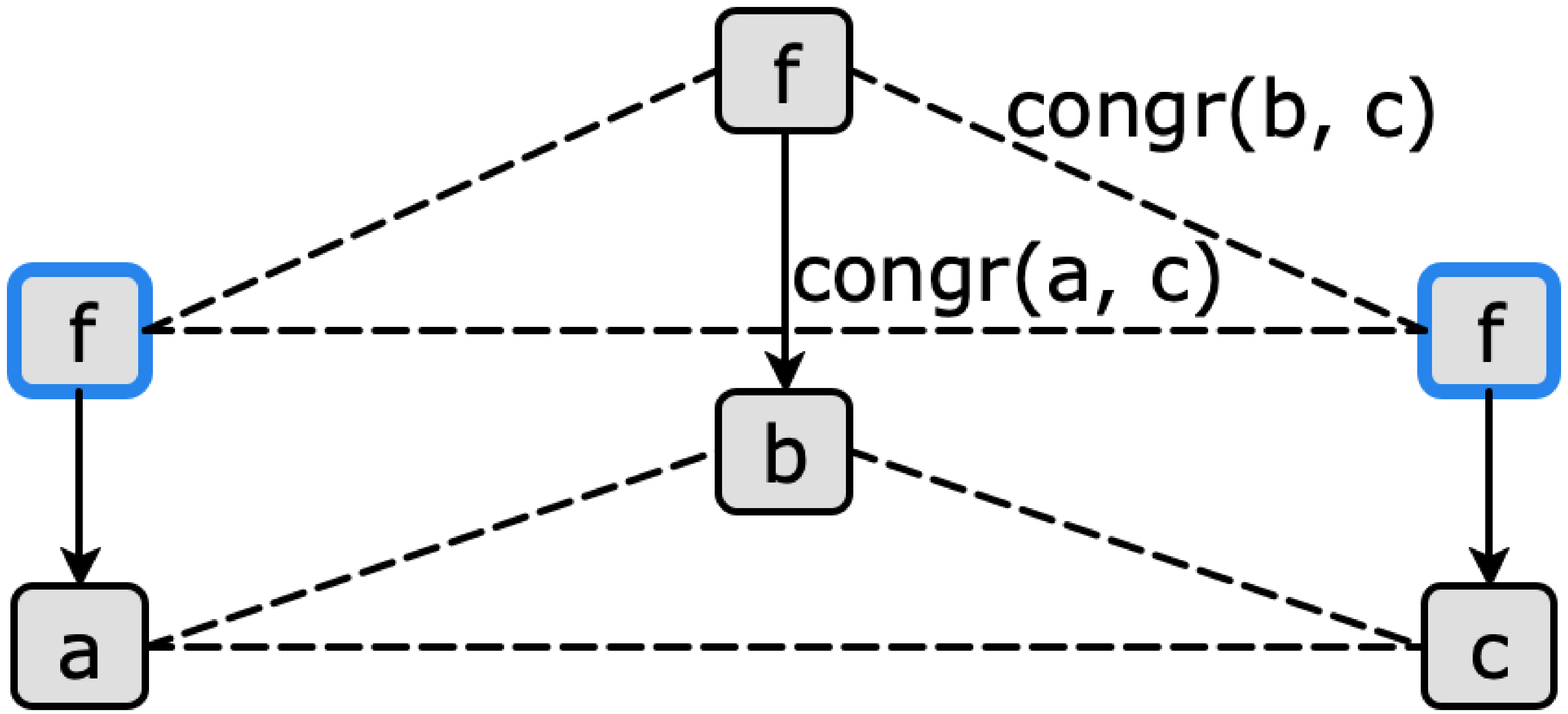
a = c useful



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Proof size: 1

Leveraging Additional Equalities

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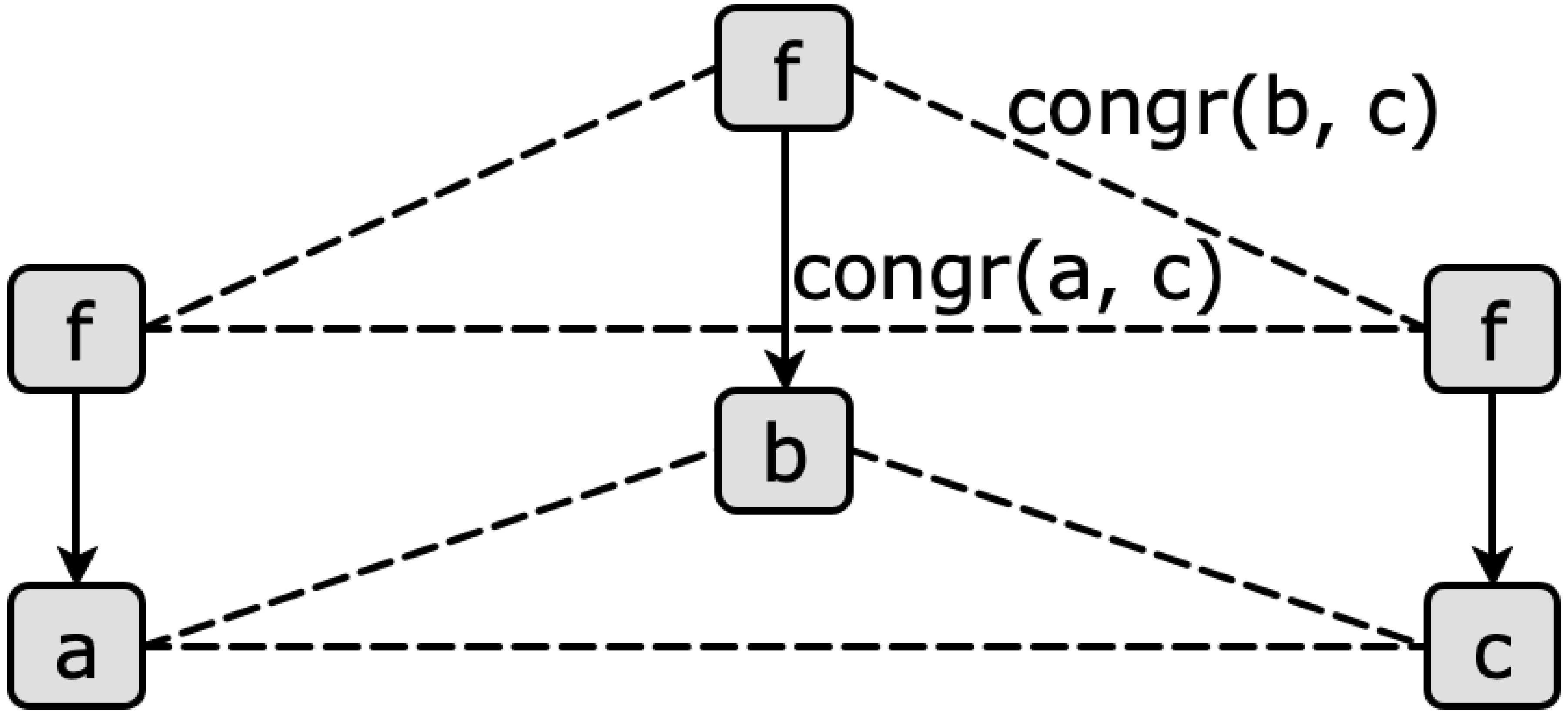

Prove f(a) and f(c) are equal:

Prove f(a) = f(c) by congruence:

a = c

done!

Proof size: 1

Leveraging Additional Equalities

Inputs:

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f(a) = f(b)

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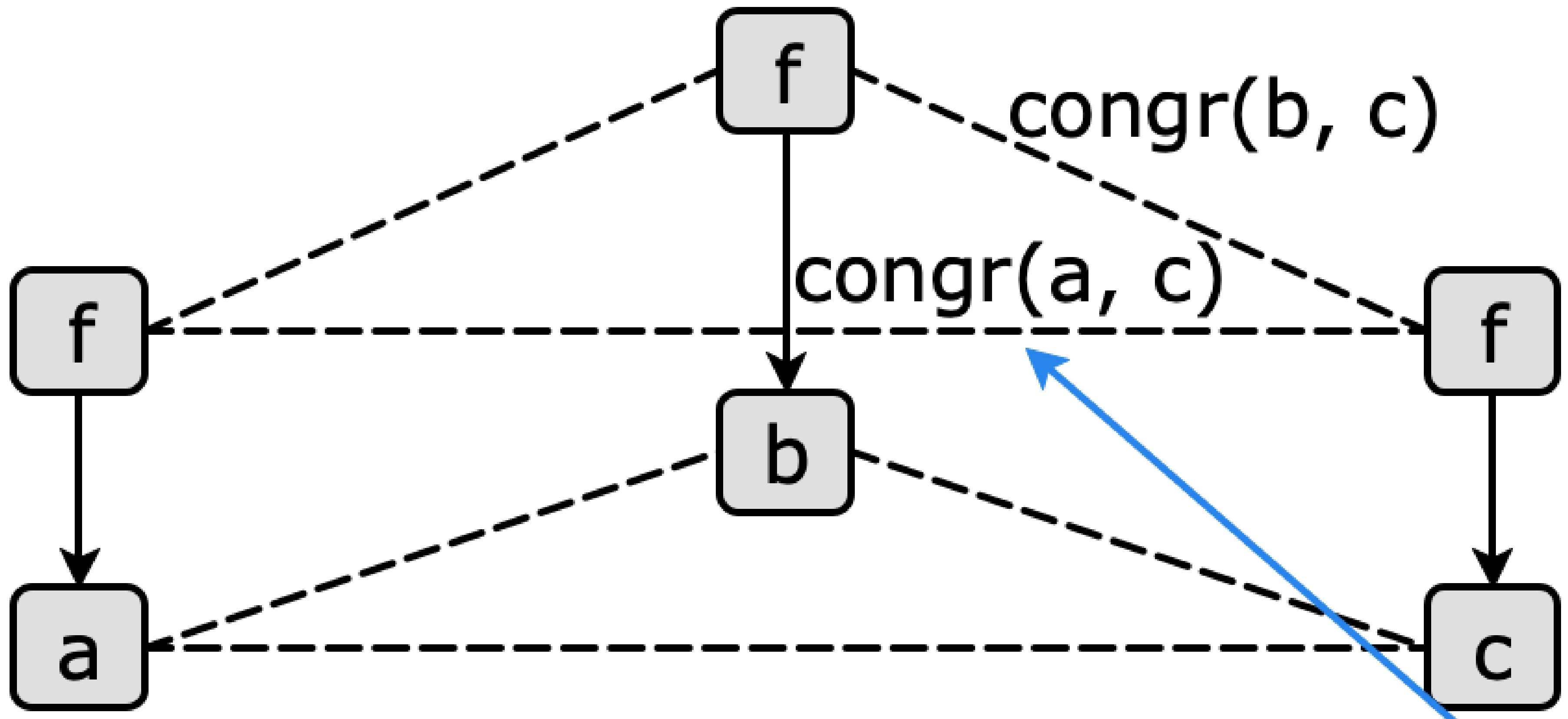
b = c

a = c useful



Prove f(a) and f(c) are equal:

Prove f(a) = f(c) by congruence:

a = c

done!

Proof size: 1

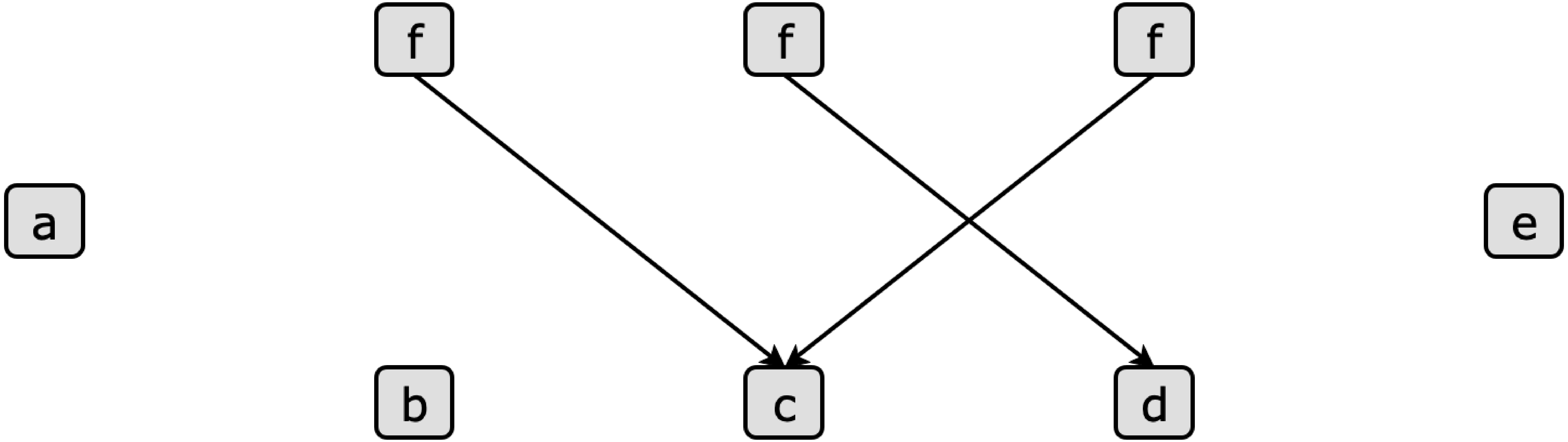
Key idea:

Try alternate path

new!

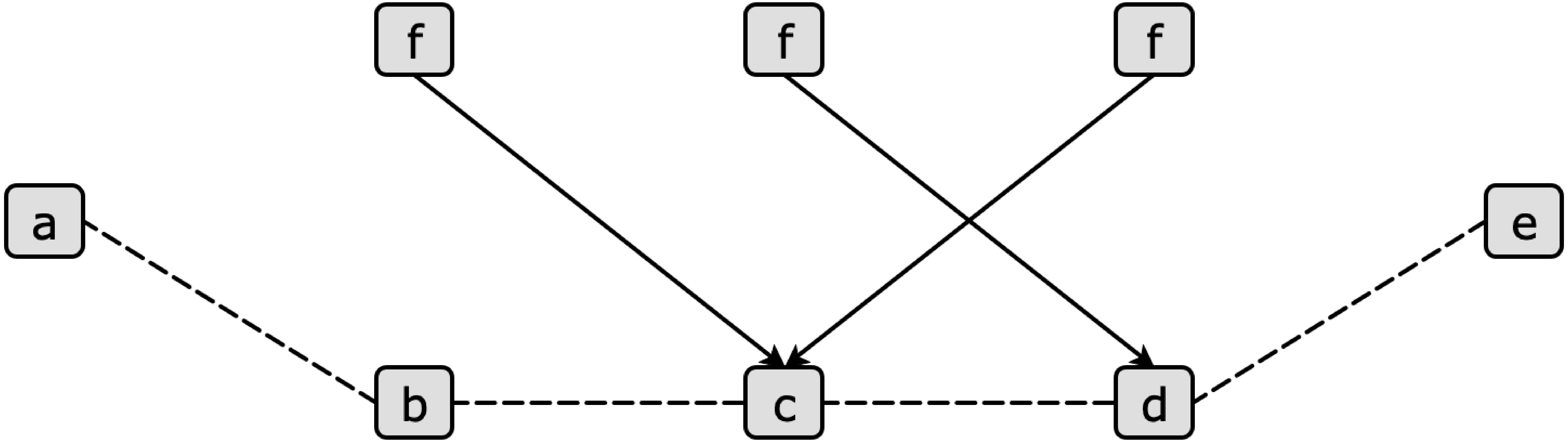
The Crux of The Problem

Prove a = e



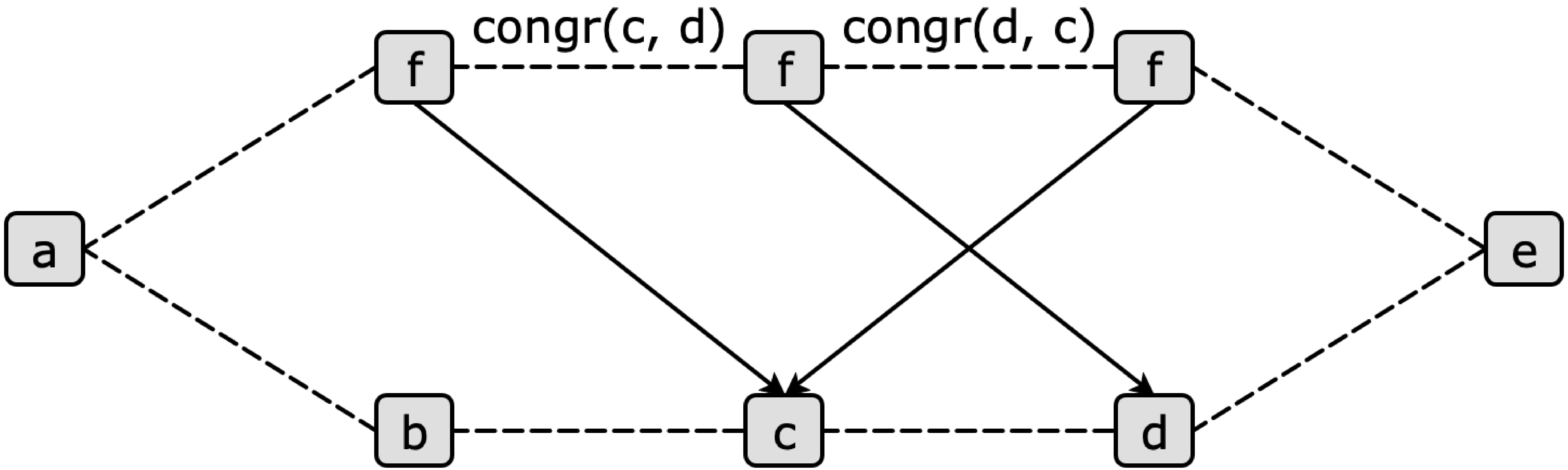
The Crux of The Problem

Prove a = e



The Crux of The Problem

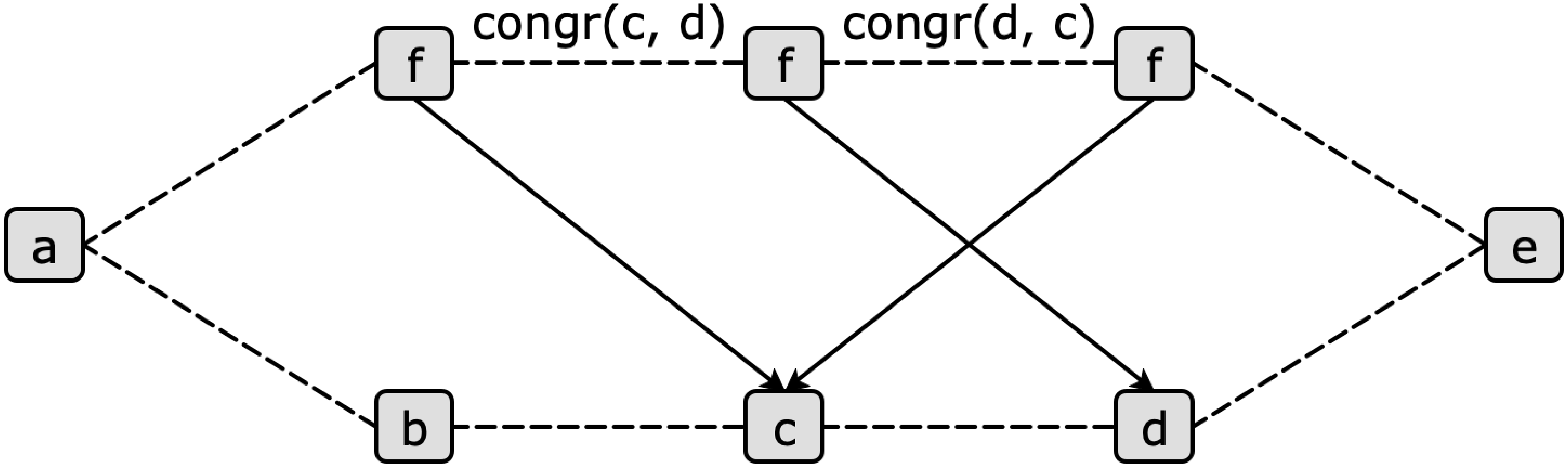
Prove a = e



The Crux of The Problem

Prove a = e

Bottom: size 4

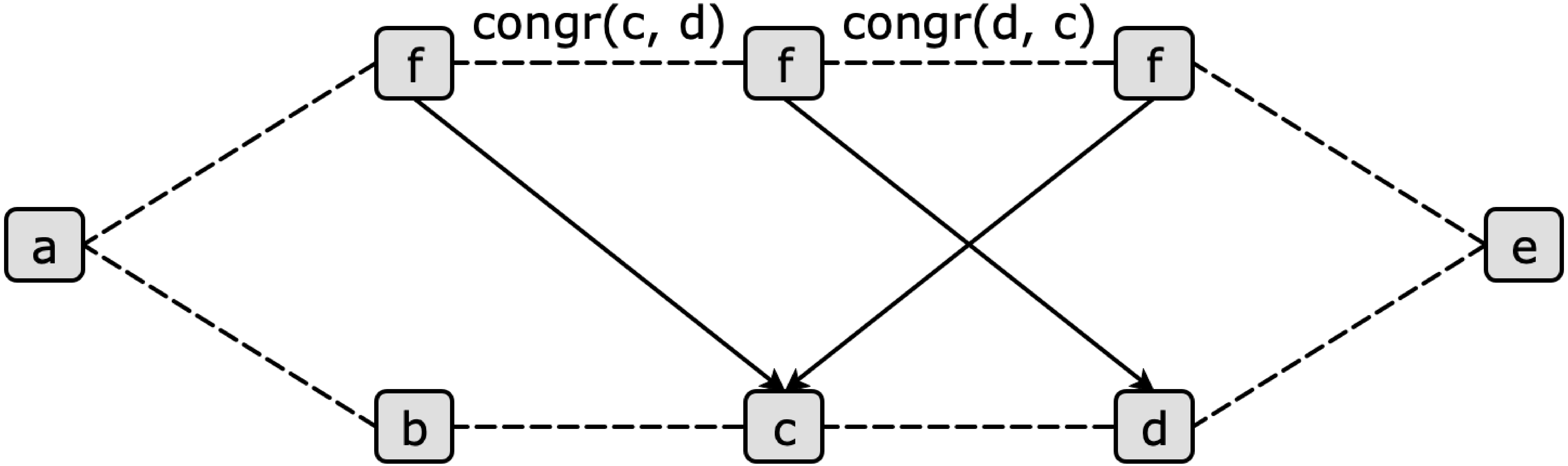


The Crux of The Problem

Prove a = e

Bottom: size 4

Top: ???

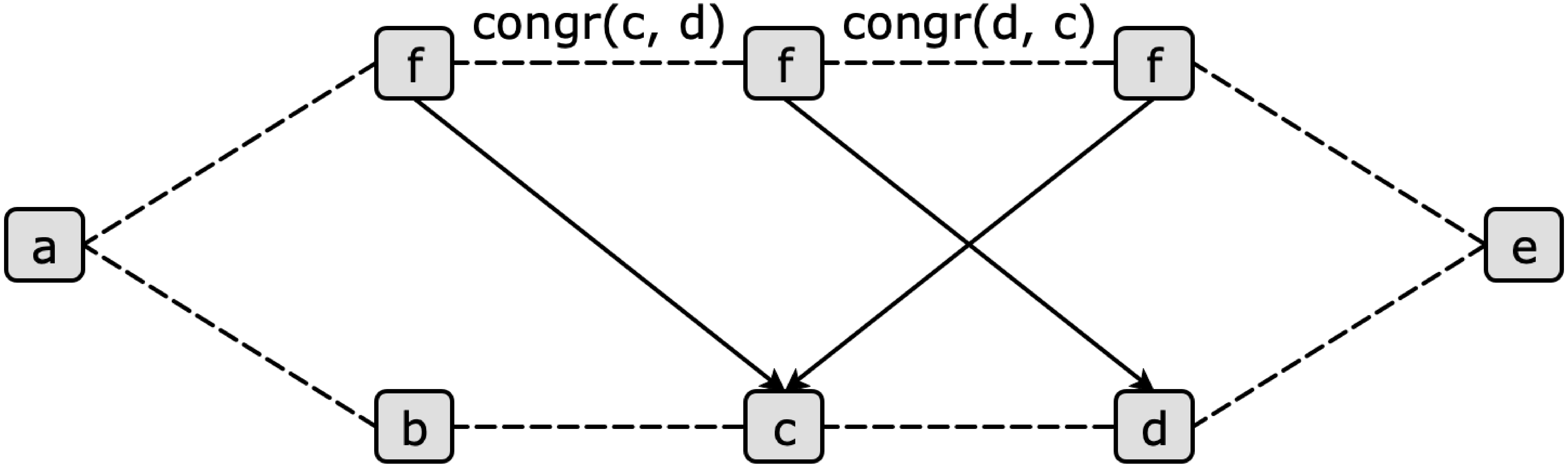


The Crux of The Problem

Prove a = e

Bottom: size 4

Top: size 3



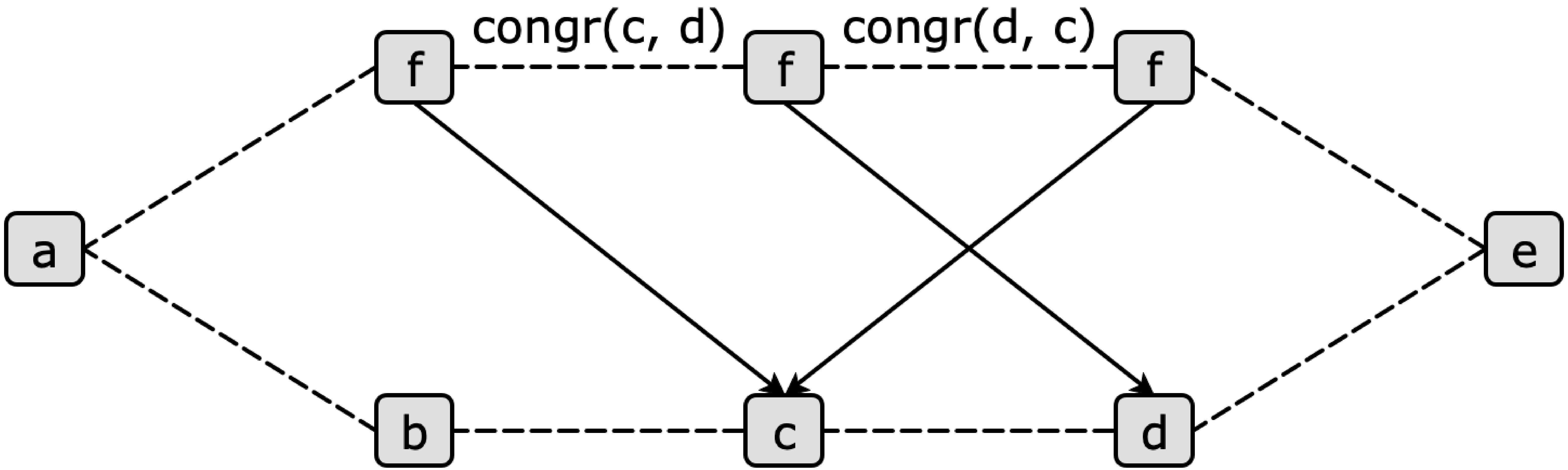
The Crux of The Problem

Prove a = e

Bottom: size 4

Top: size 3

Proof re-use is free

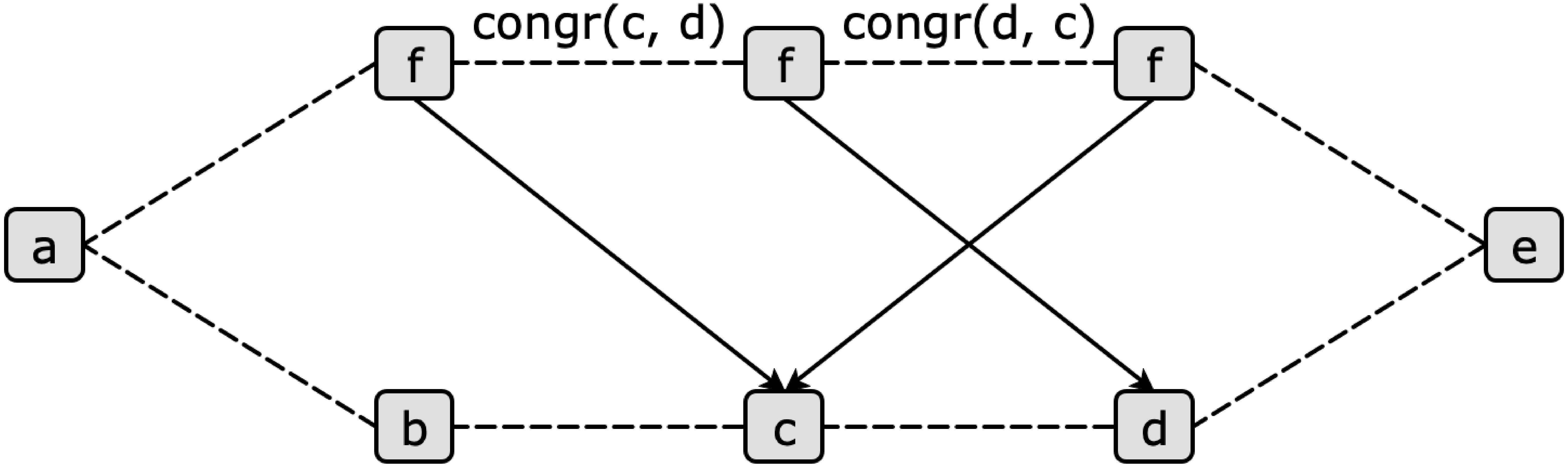


The Crux of The Problem

Prove a = e

Bottom: size 4

Top: size 3



Proof re-use is free

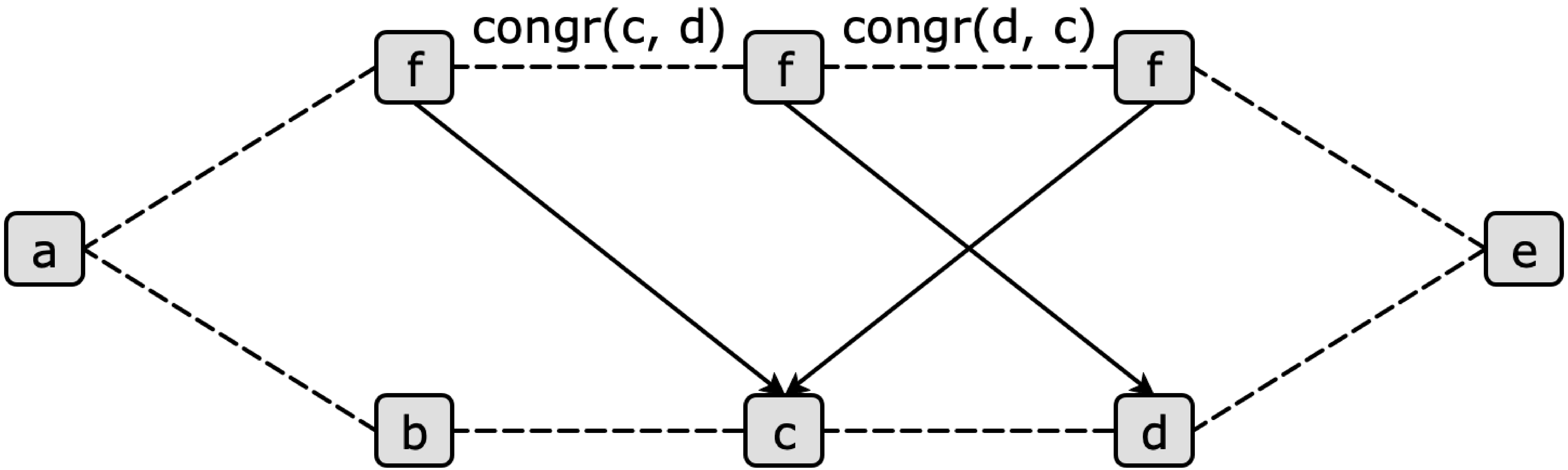
Subproofs can be complex

The Crux of The Problem

Prove a = e

Bottom: size 4

Top: size 3



Proof re-use is free

Subproofs can be complex

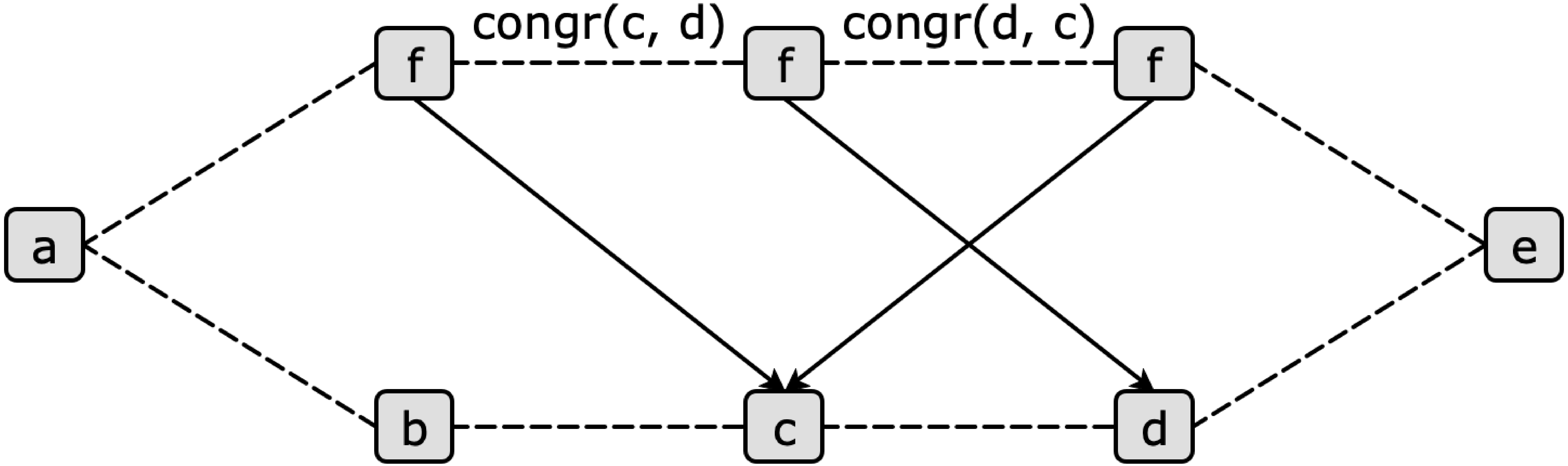
Exponential paths in graph

The Crux of The Problem

Prove a = e

Bottom: size 4

Top: size 3



Proof re-use is free

Subproofs can be complex

Exponential paths in graph

NP-Hard Problem (Fellner Et al.)

- [x] Motivation
- [x] Congruence Closure
- [x] Proofs from Congruence Closure
- [ ] Finding Small Proofs

27.2% smaller proofs!

Idea: Shortest Path?

Inputs:

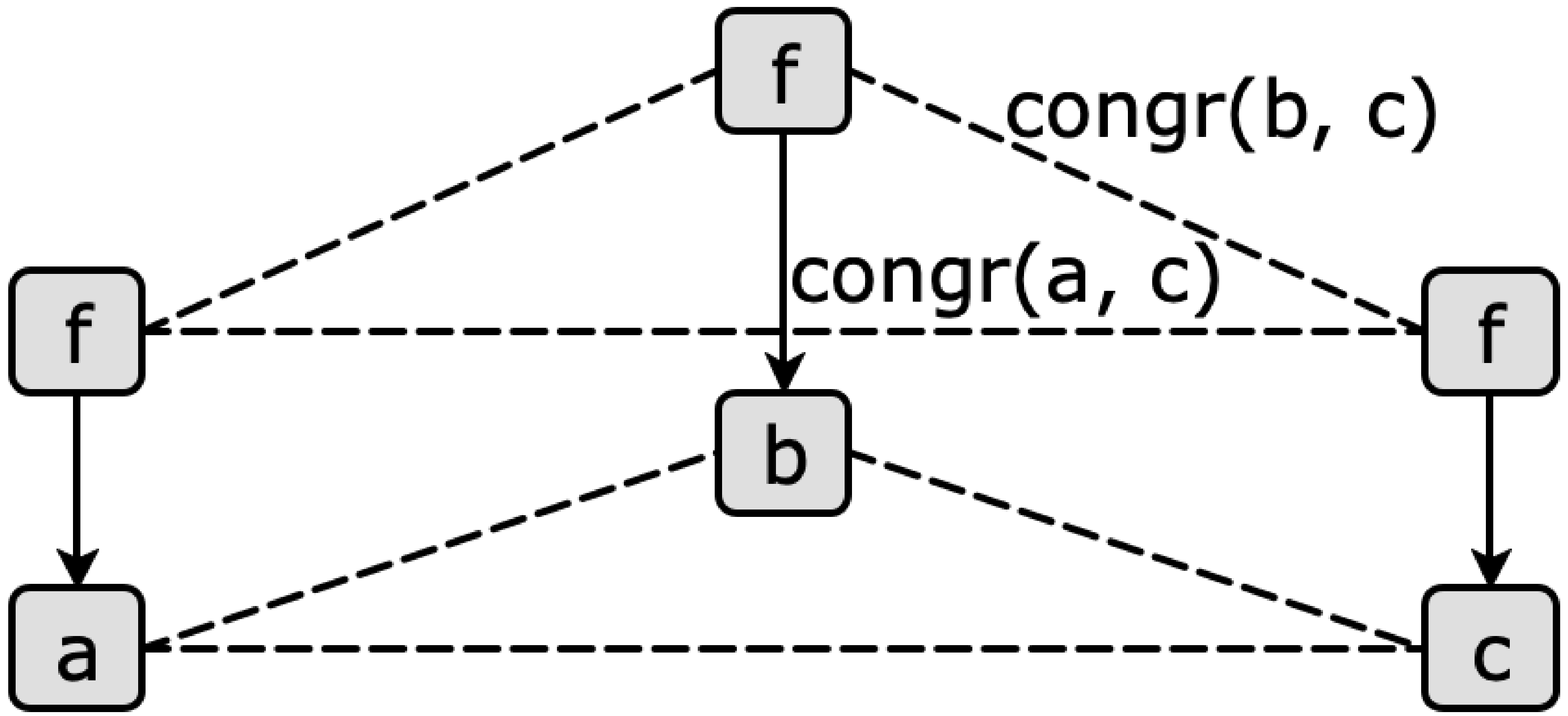
f(c)

f(a) = f(b)

a = b

b = c

a = c useful



Idea: Shortest Path?

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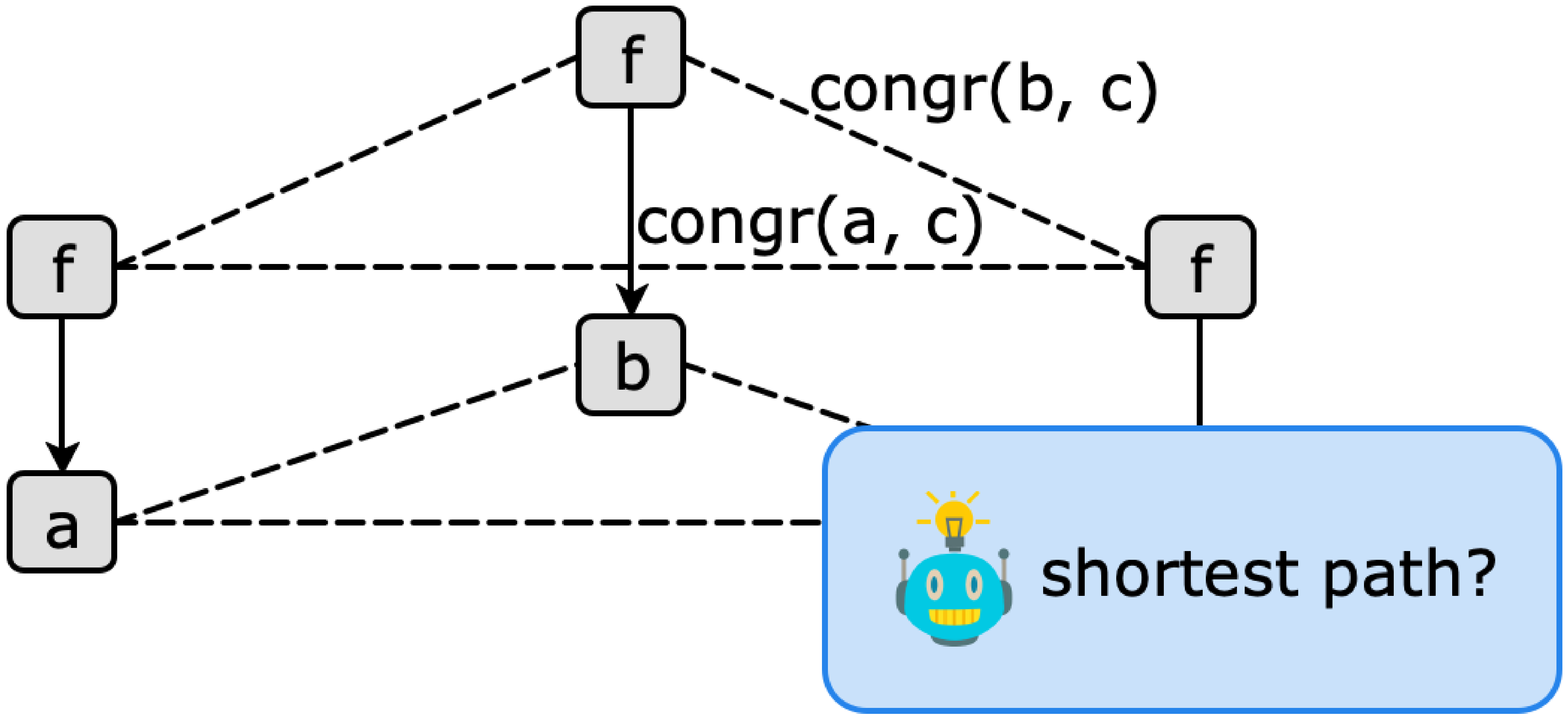

Idea: Shortest Path?

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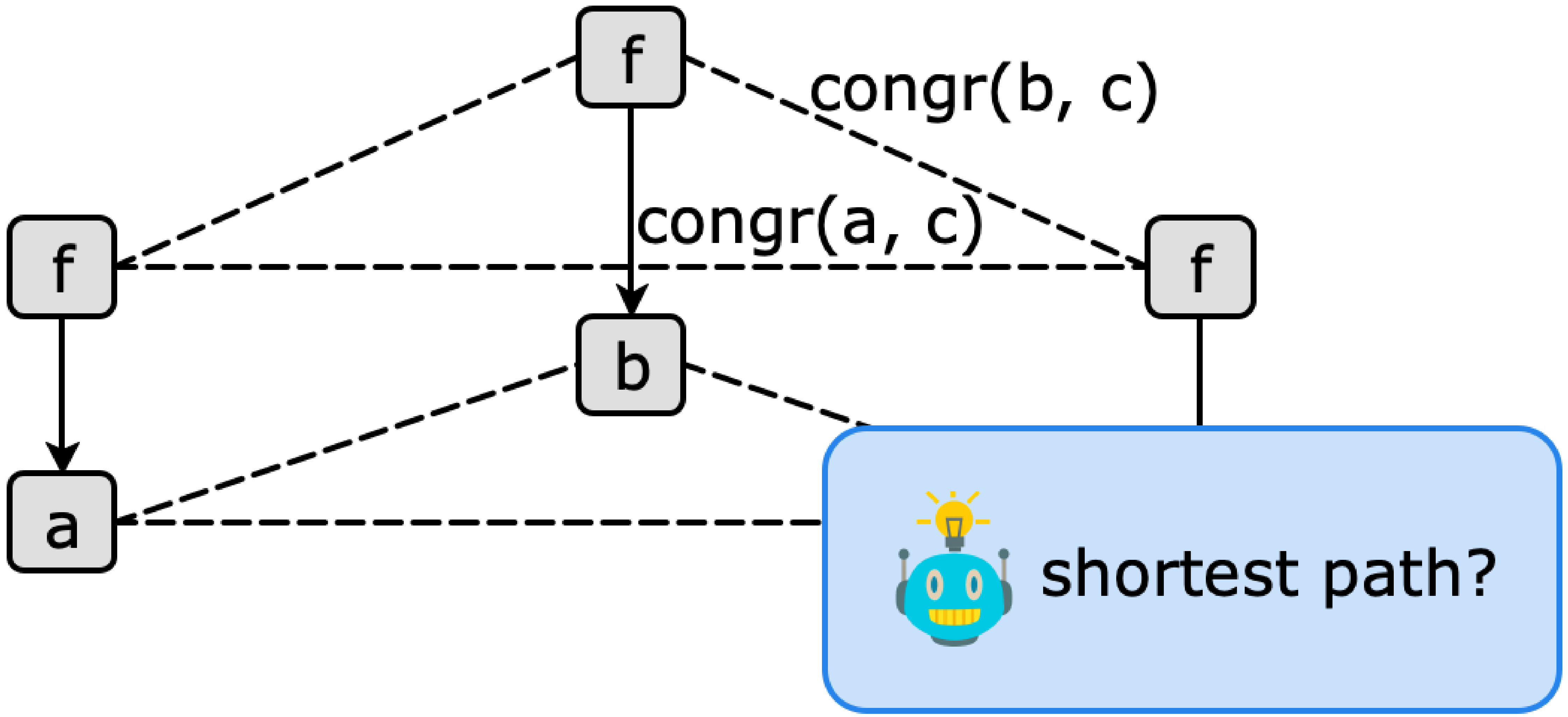


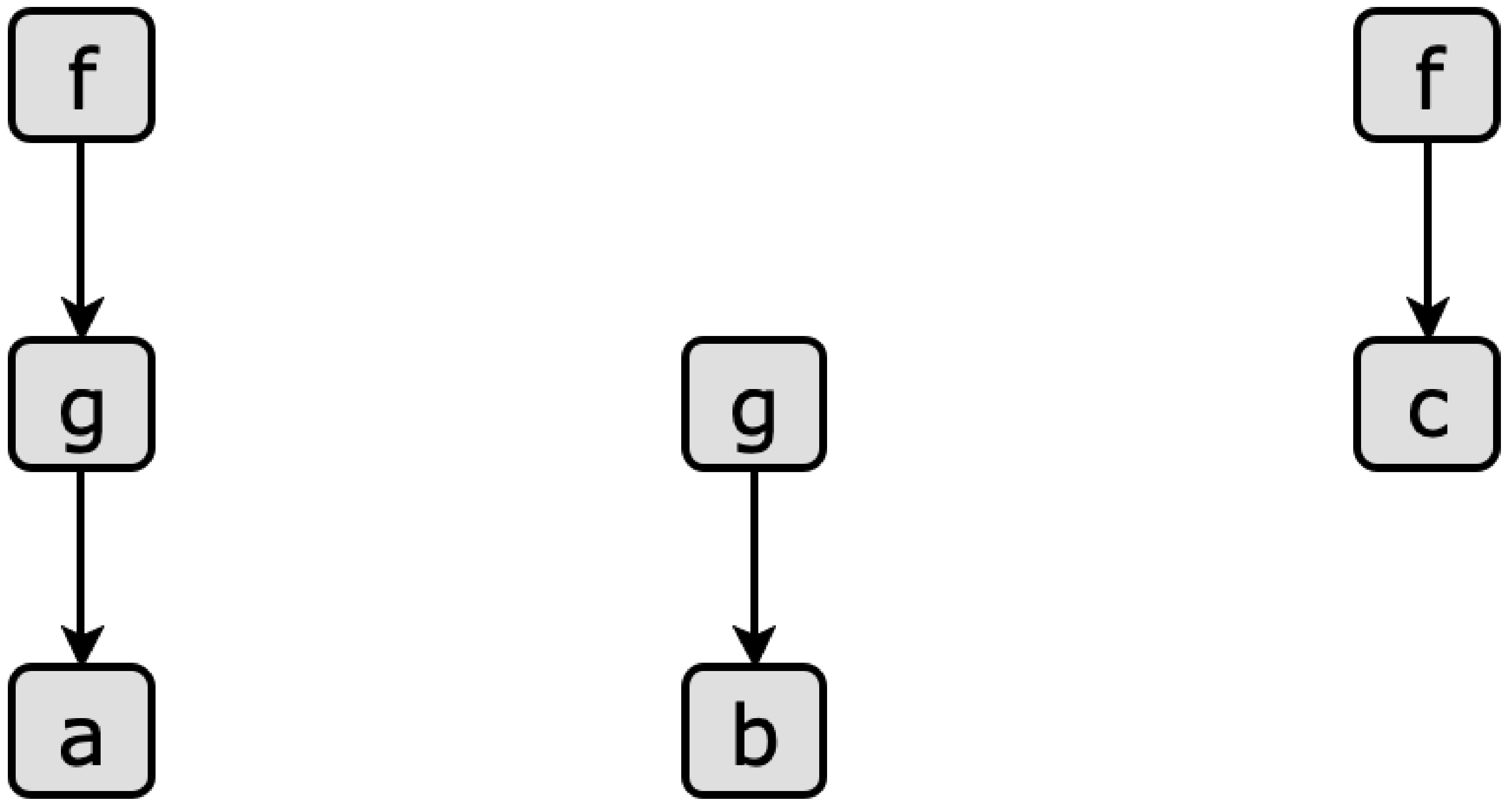
Problem: how big are congruence edges?

Proof Size Estimation

Inputs:

a = b

g(b) = c



Proof Size Estimation

Inputs:

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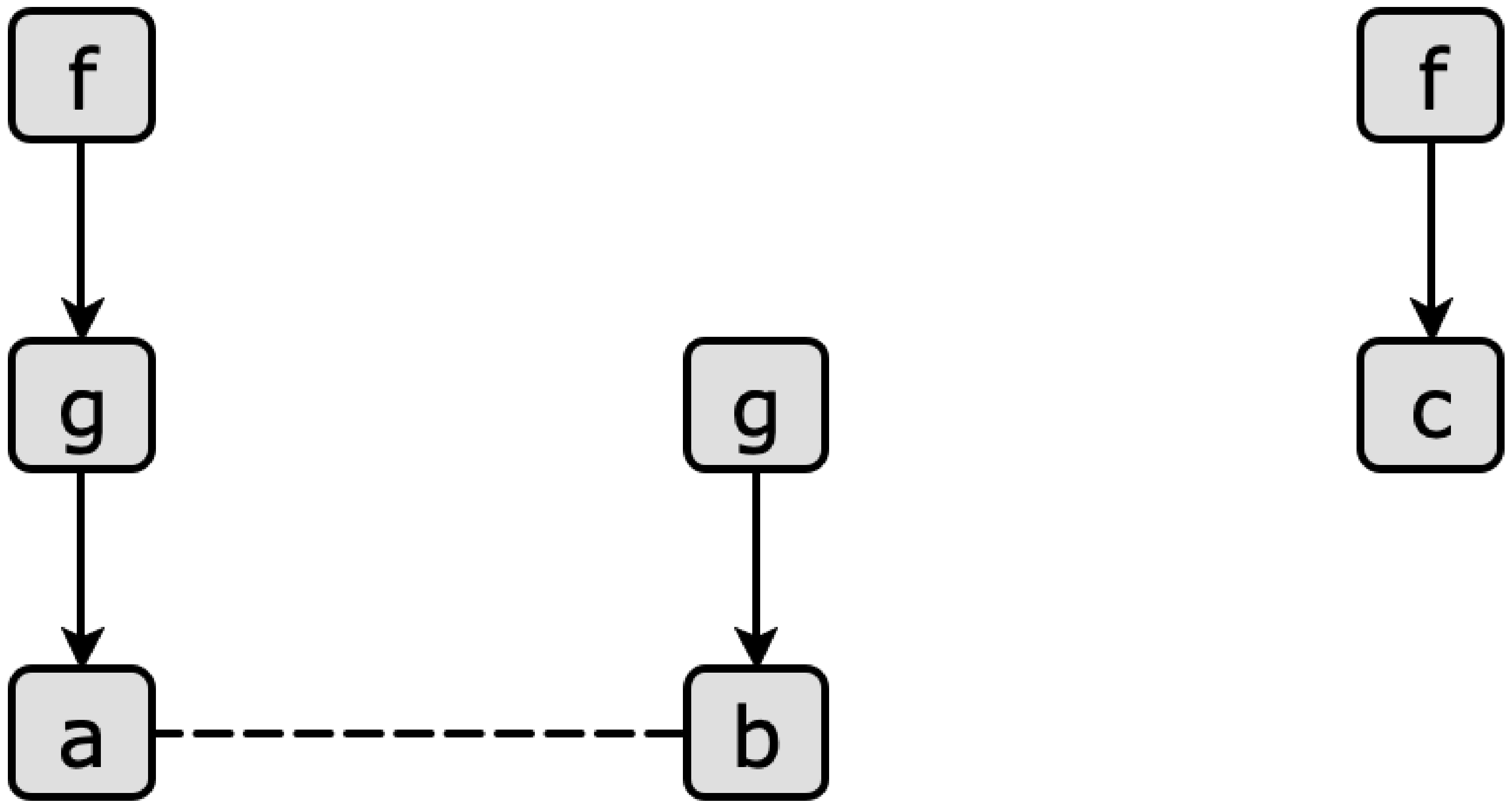
g(b) = c



Proof Size Estimation

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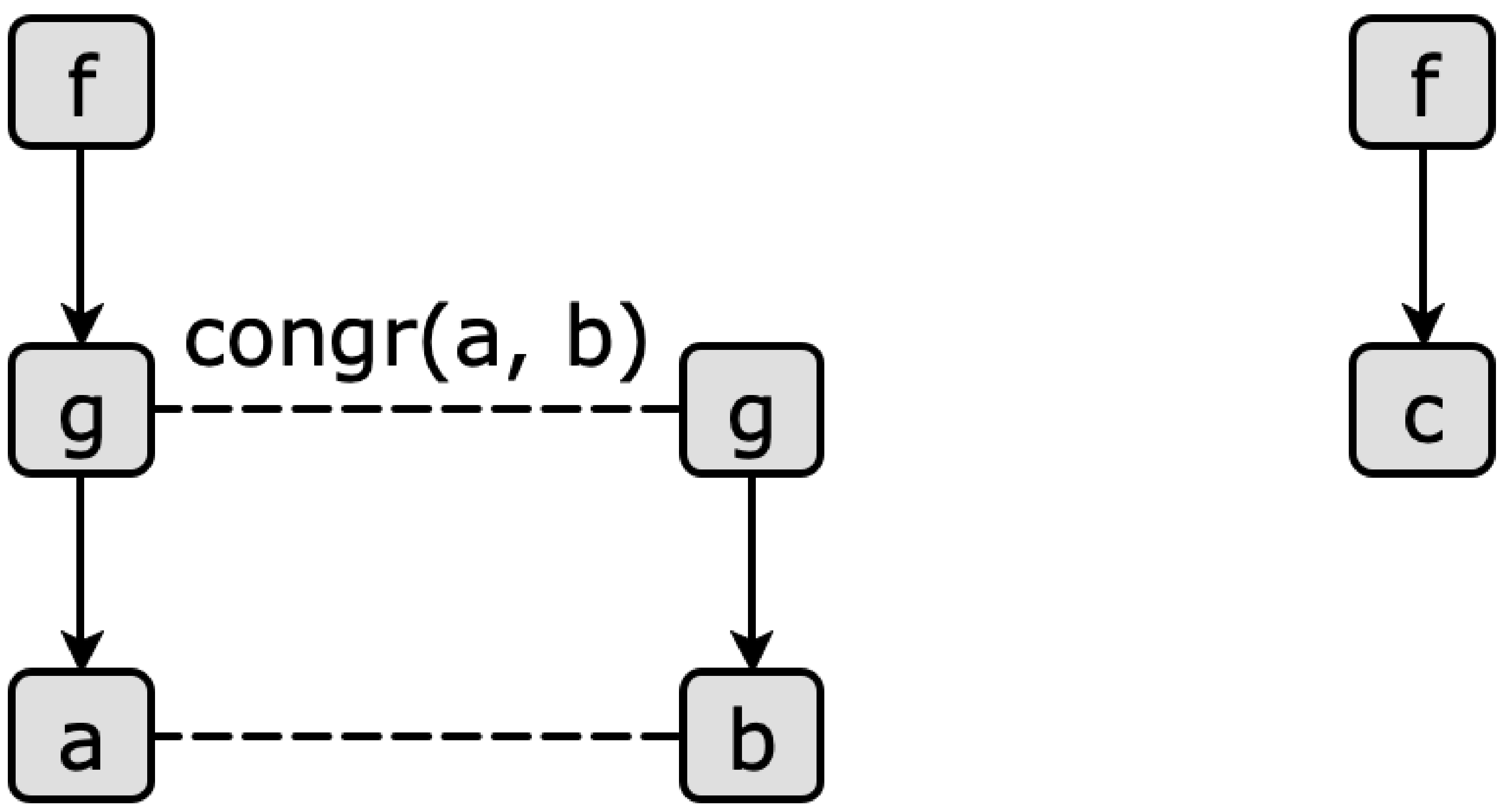

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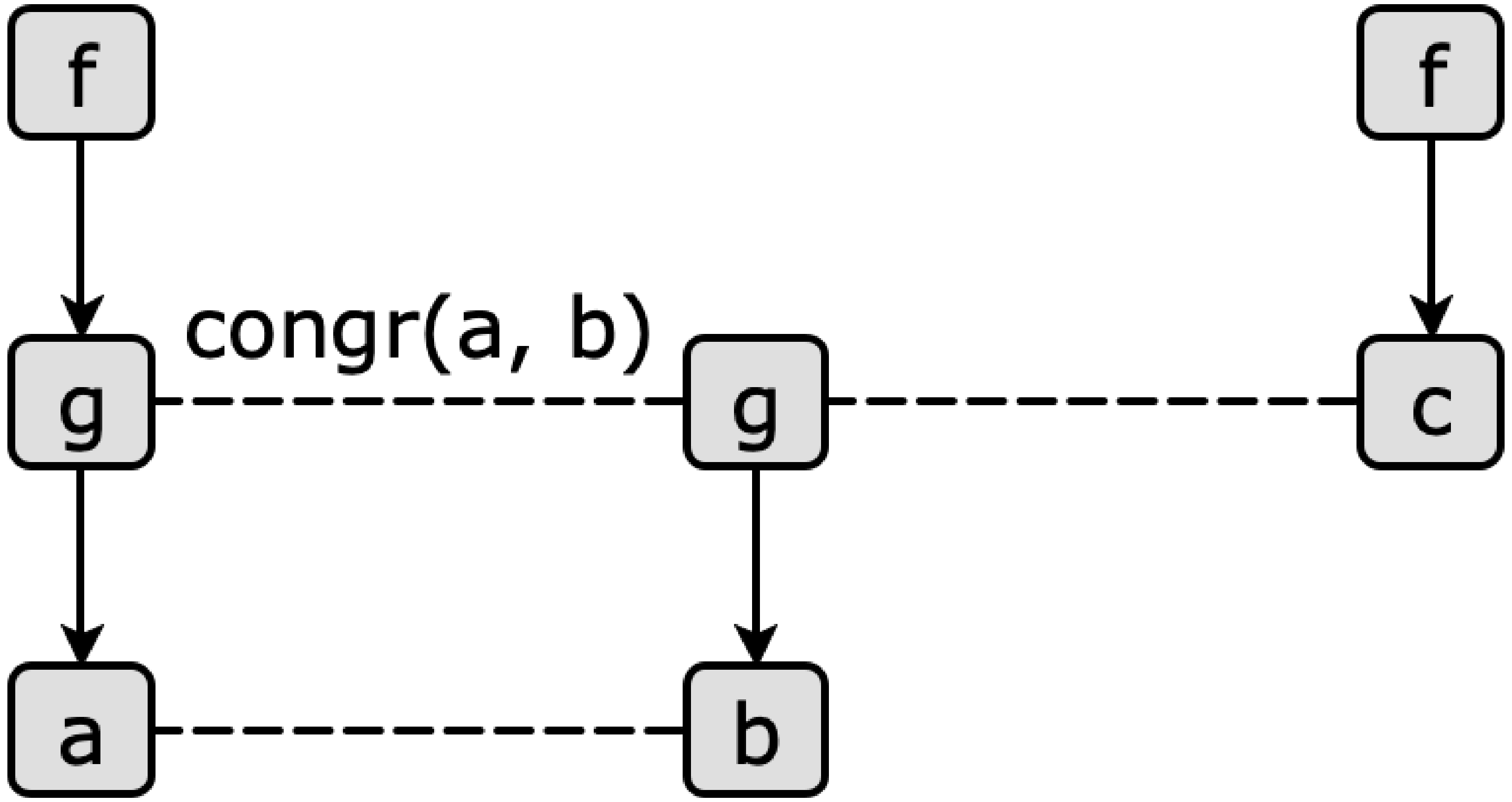

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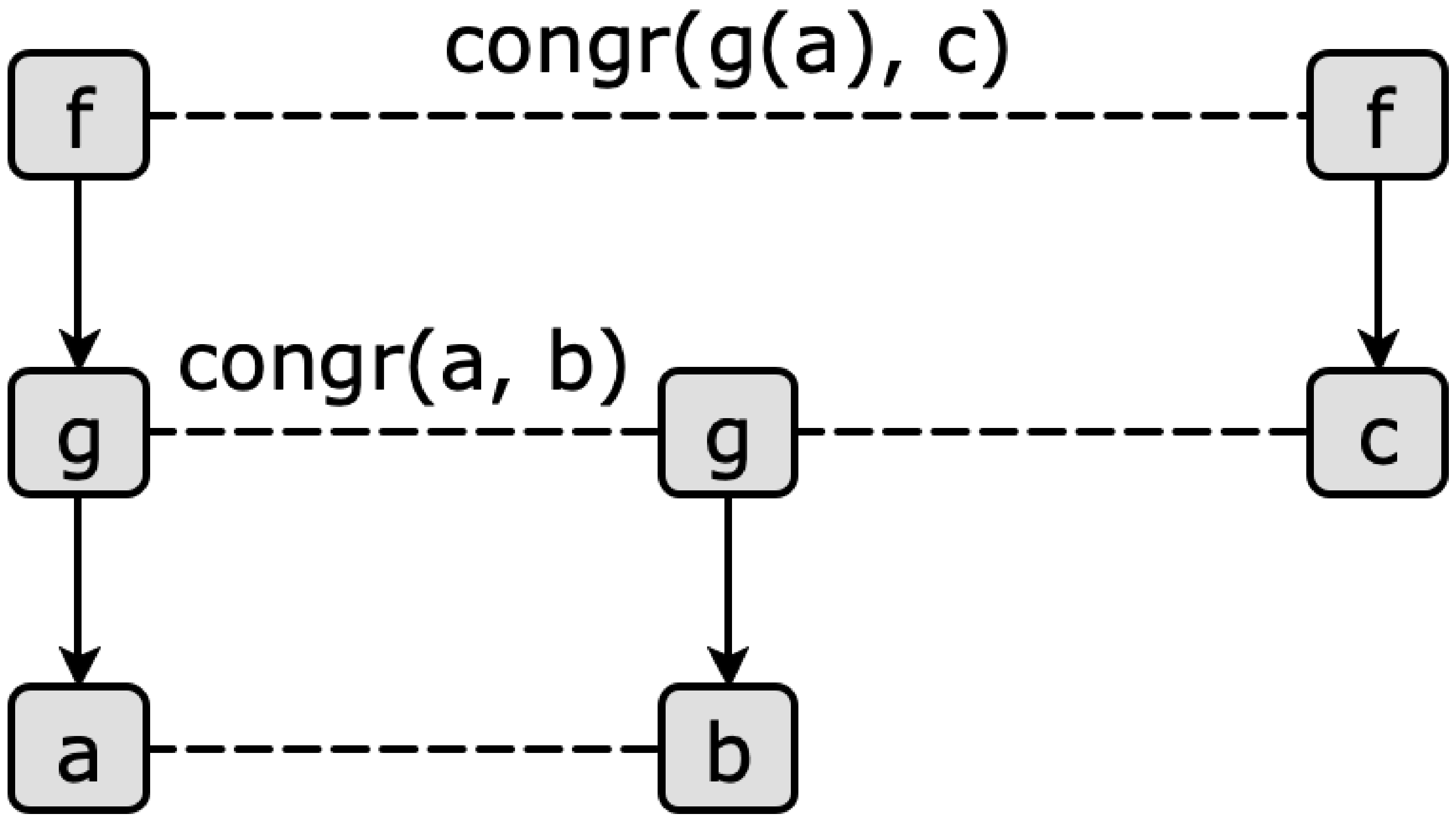

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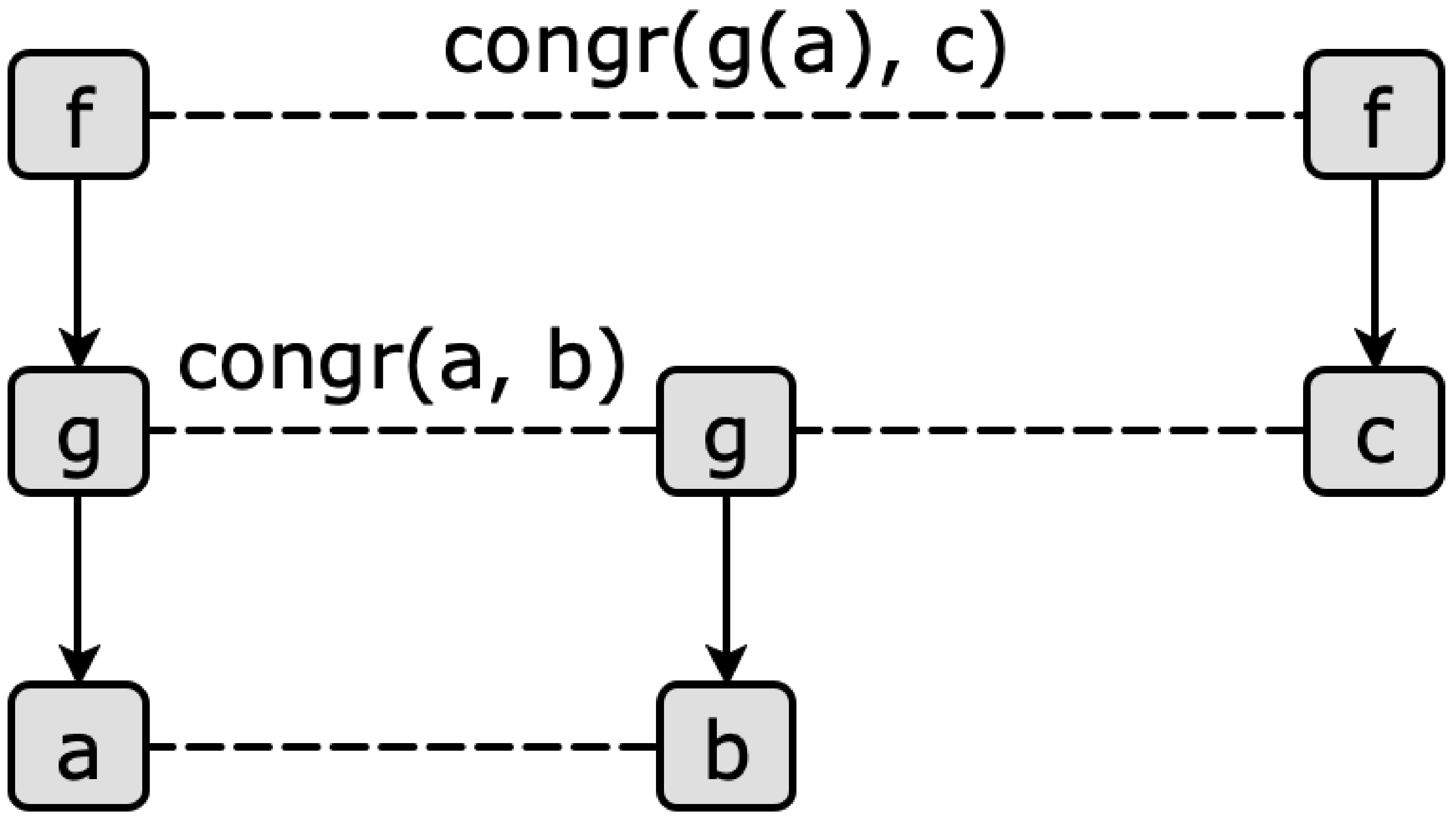


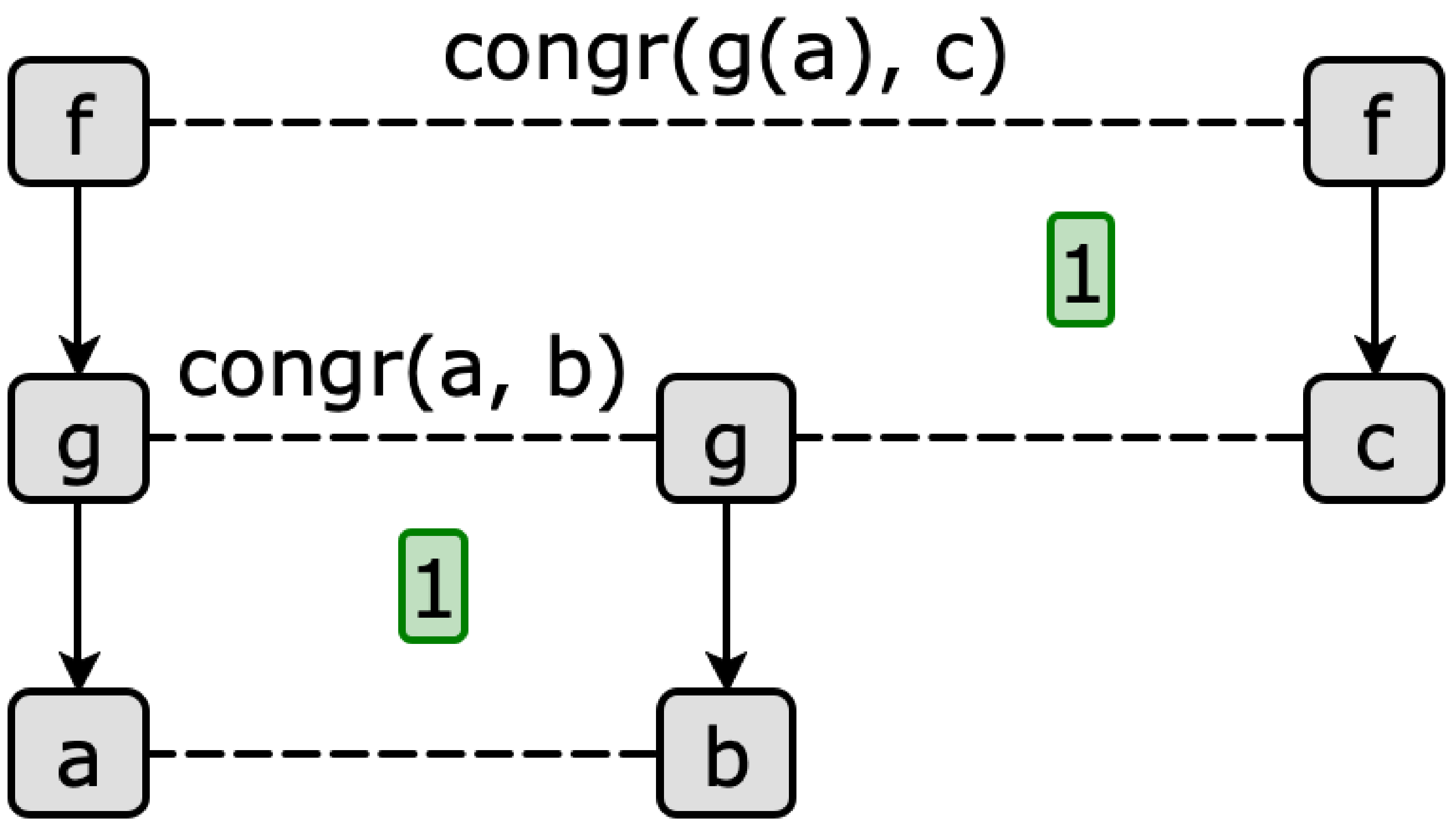
Key idea: compute estimates bottom-up

Proof Size Estimation

Inputs:

a = b

g(b) = c



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Proof Size Estimation

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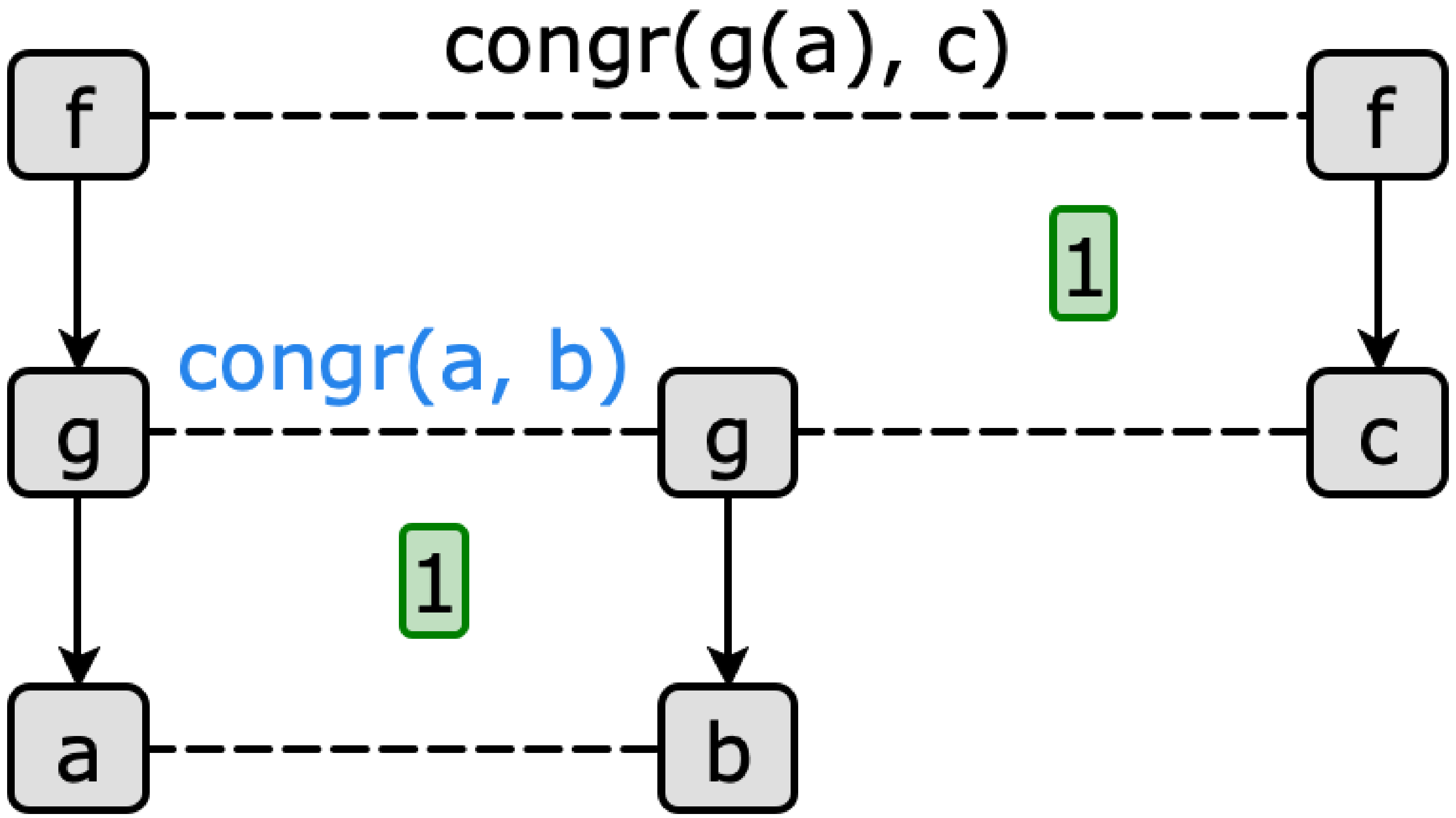

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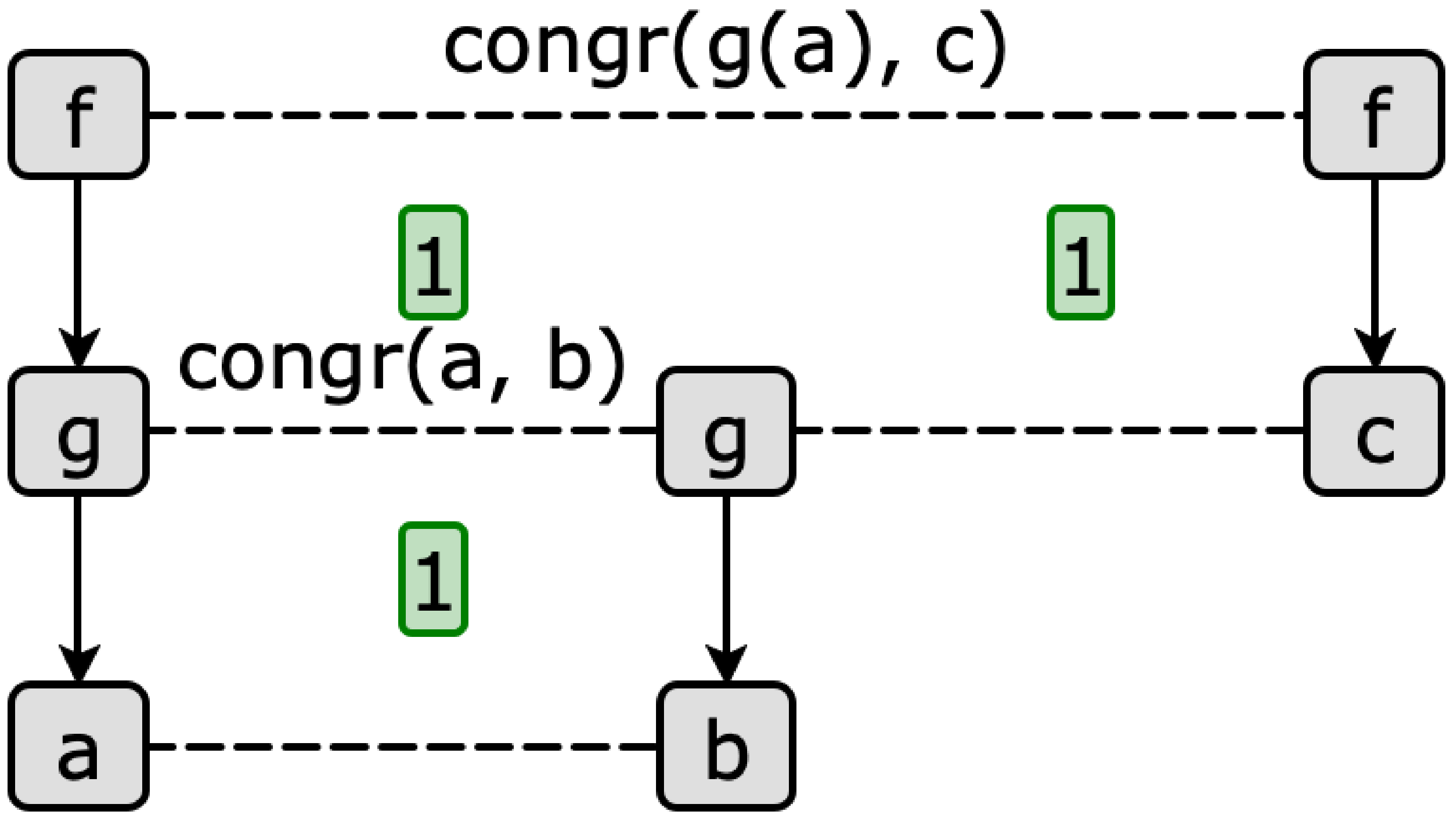

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Proof Size Estimation

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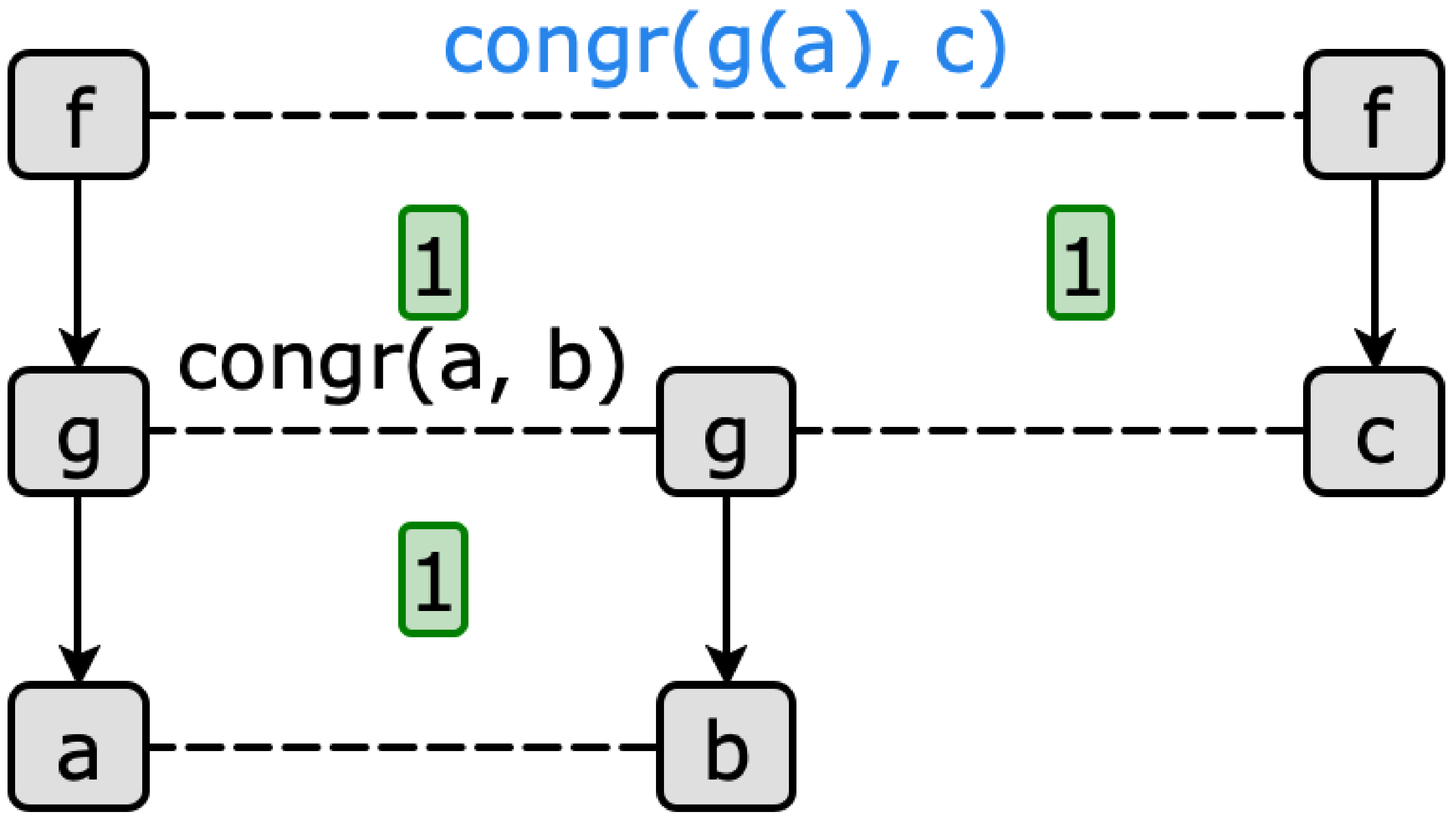

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Proof Size Estimation

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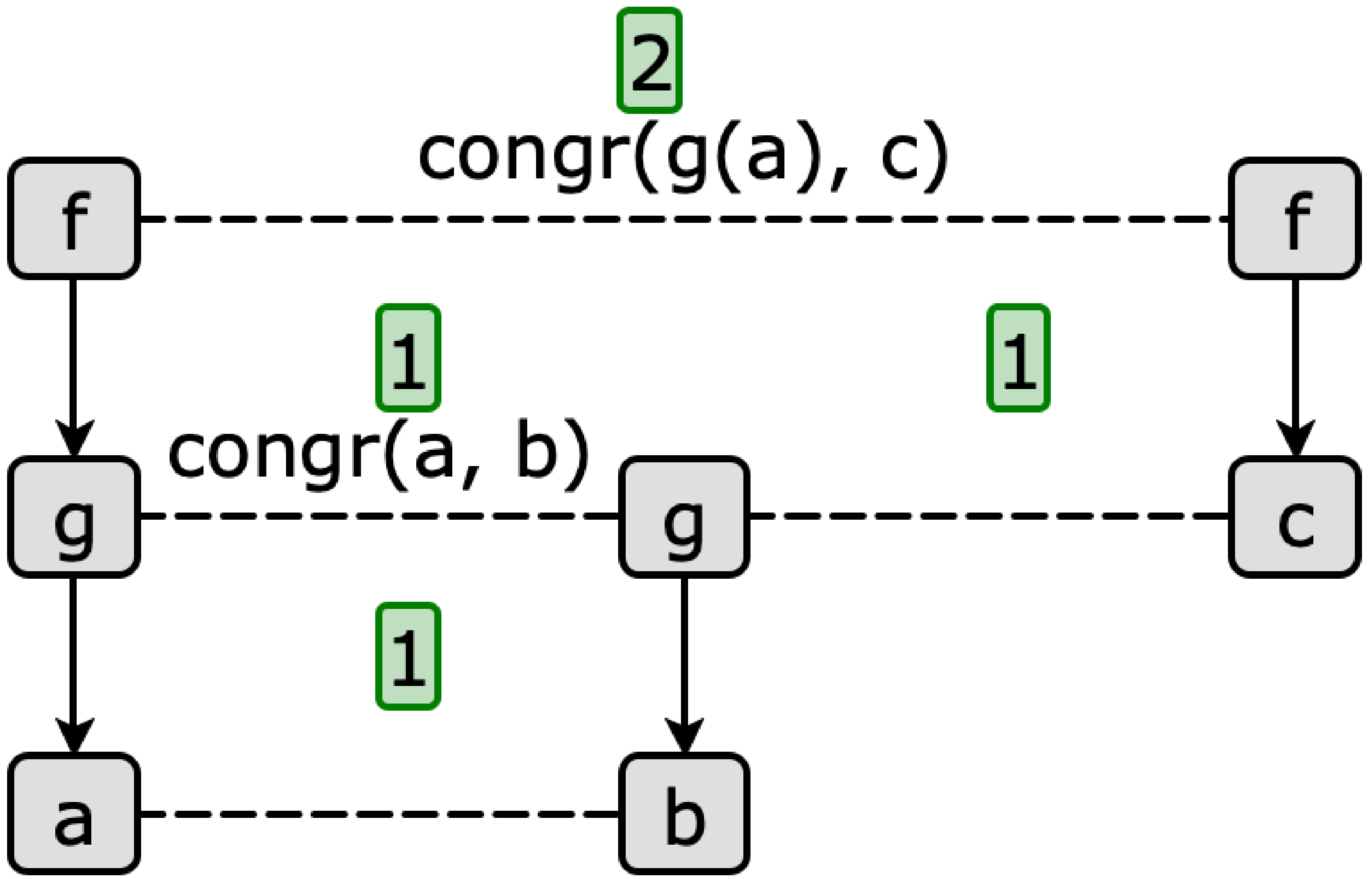

Key idea: compute estimates bottom-up

Proof Size Estimation

Inputs:

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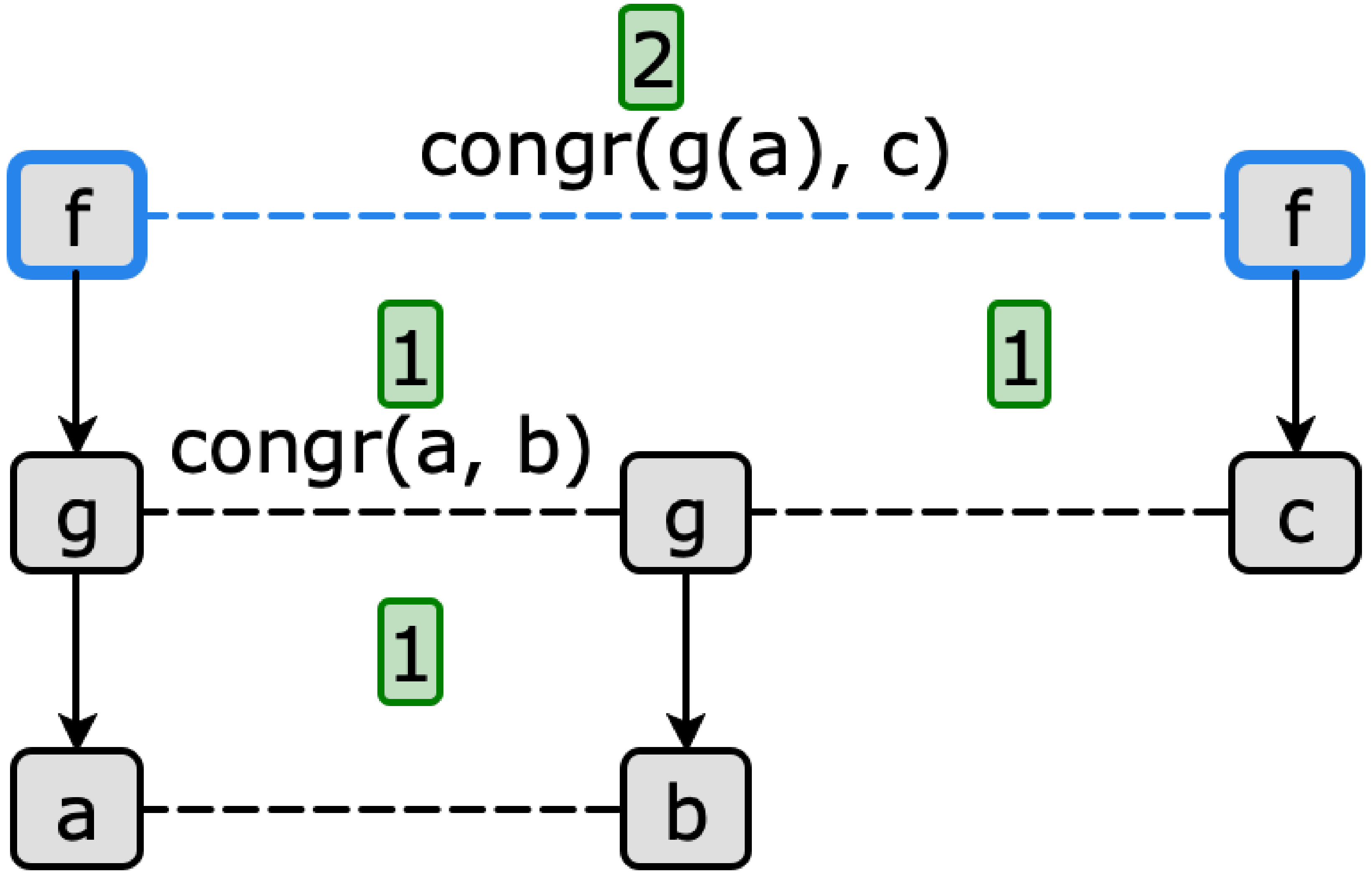
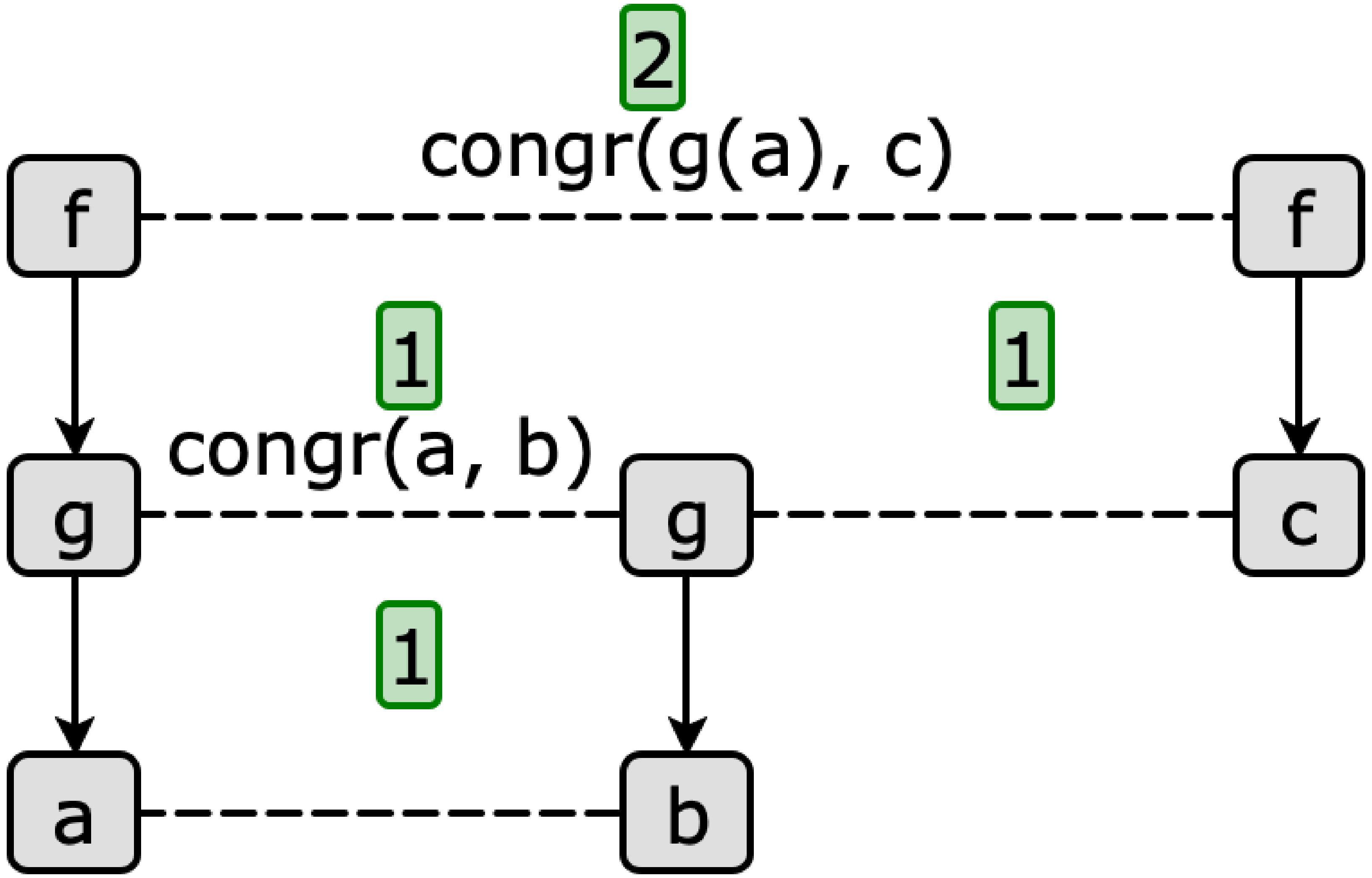
g(b) = c



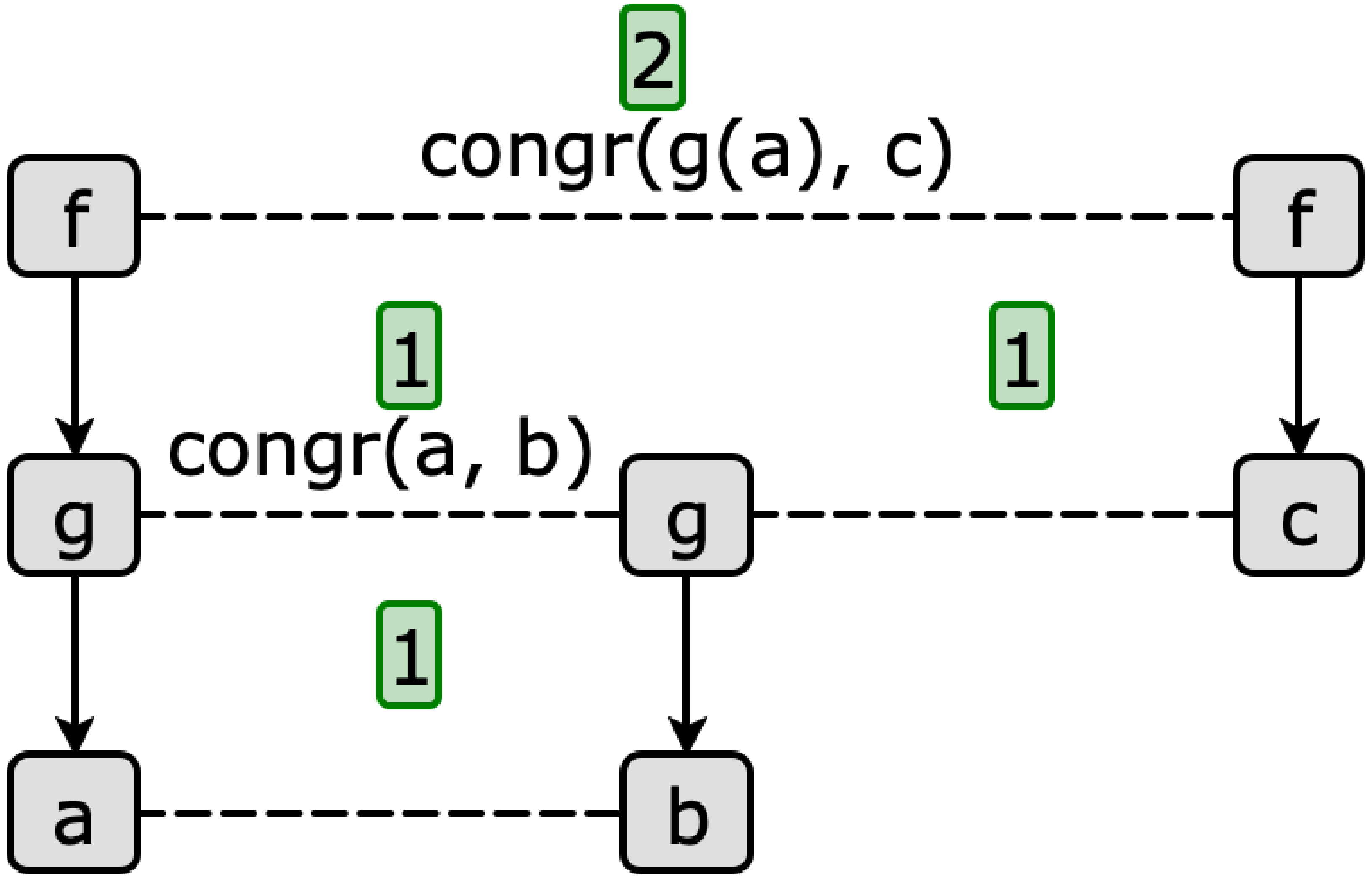
Key idea: compute estimates bottom-up

Putting it All Together



Putting it All Together

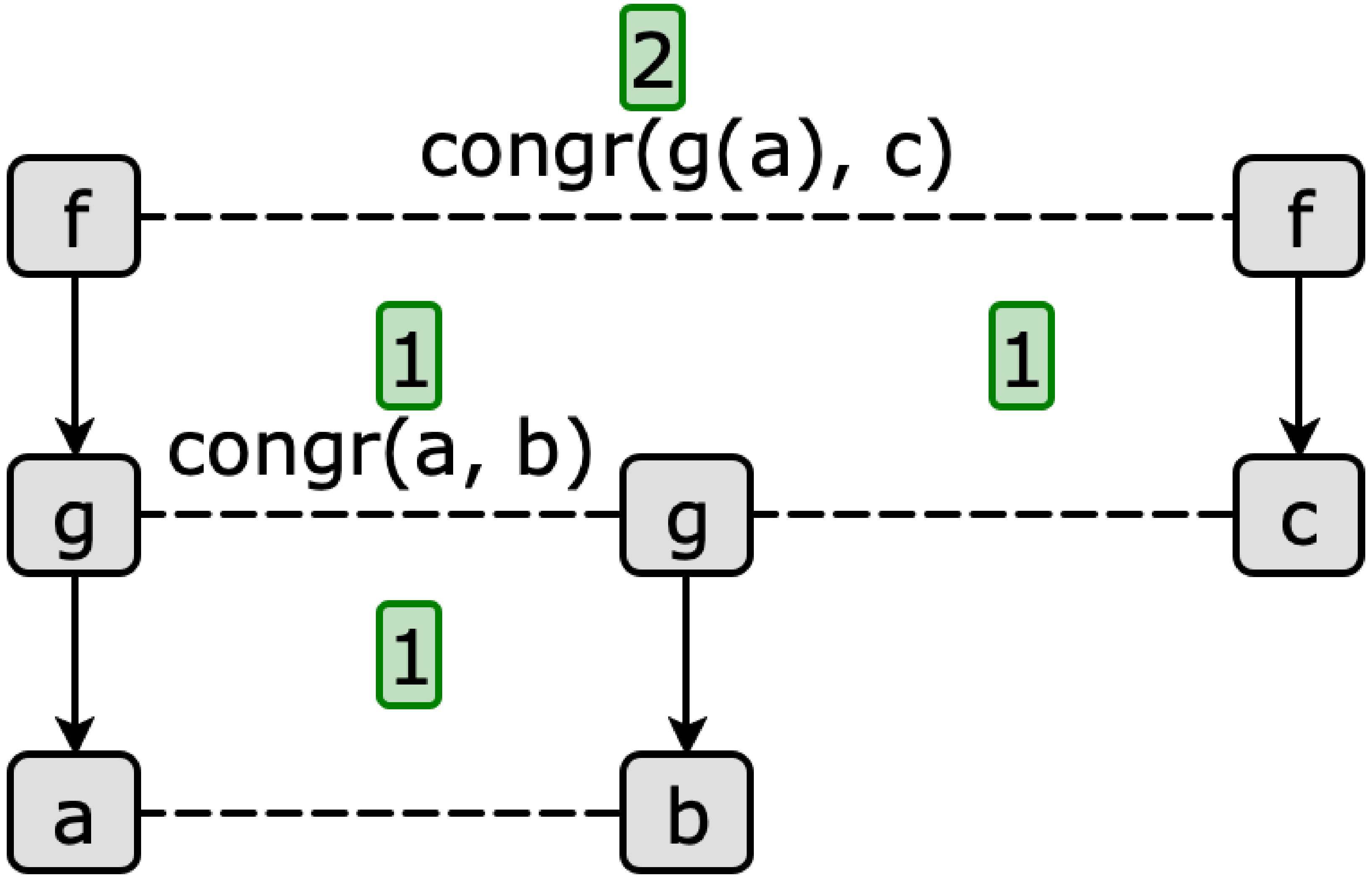
1. Compute size estimates



Putting it All Together

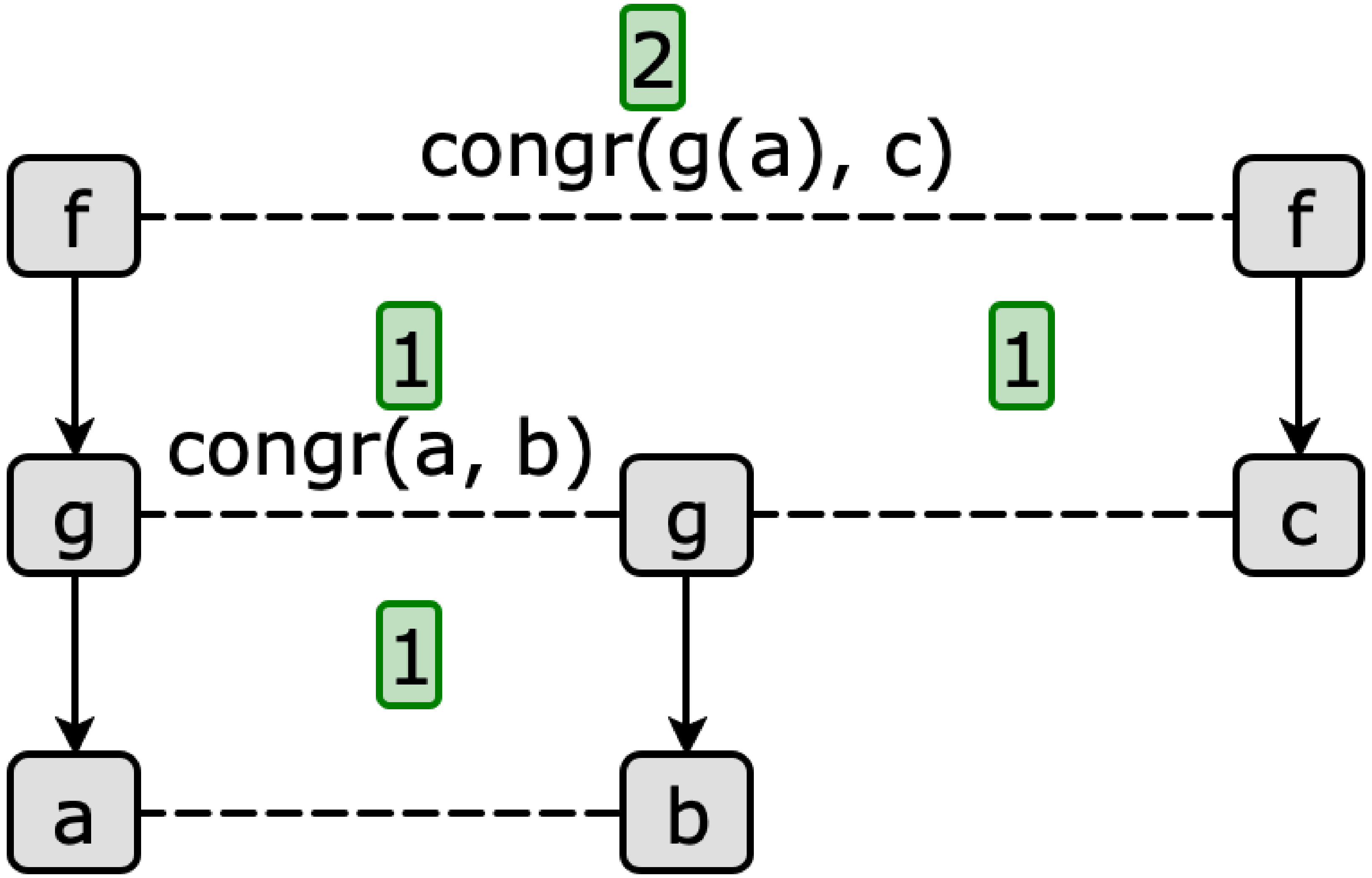
1. Compute size estimates

2. Find shortest path

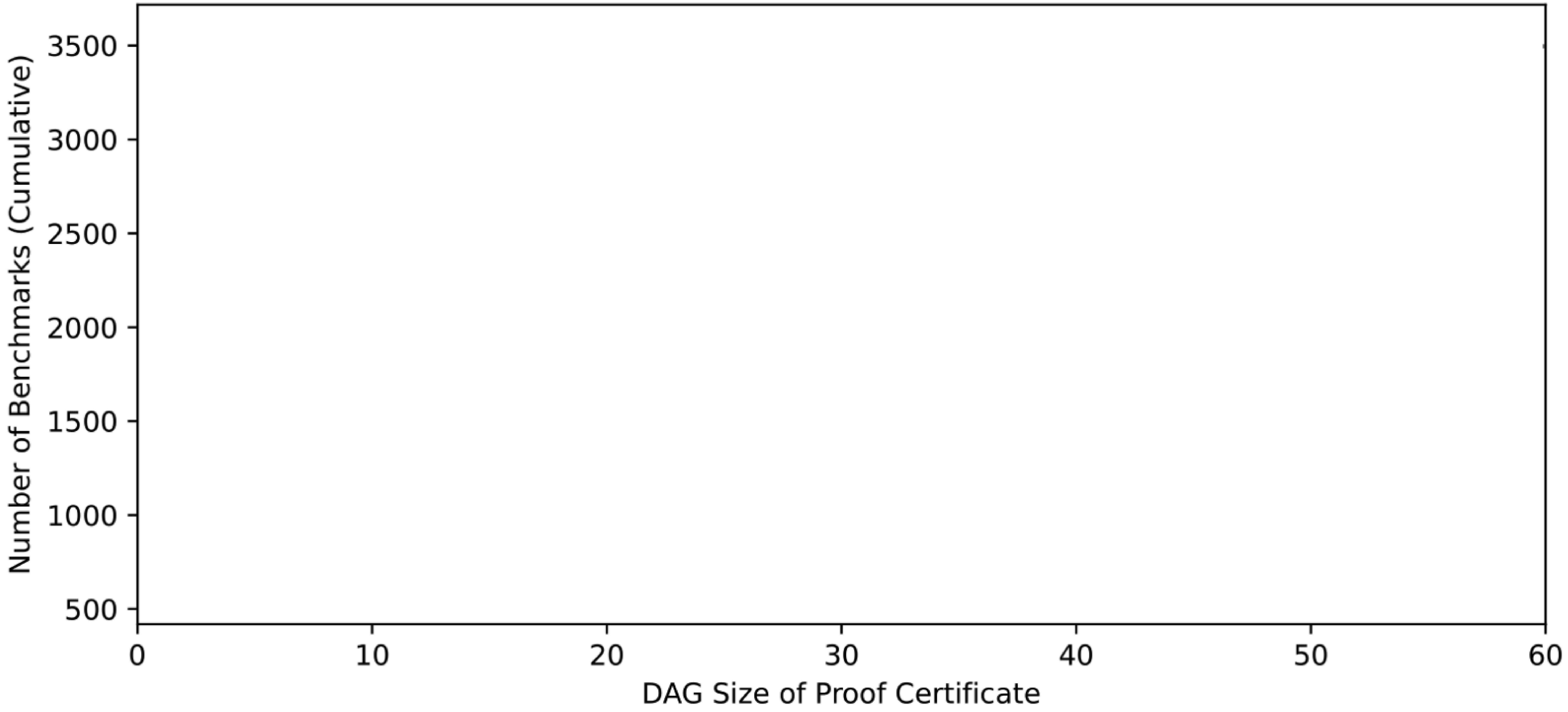


Putting it All Together

1. Compute size estimates
2. Find shortest path
3. Output extracted proof

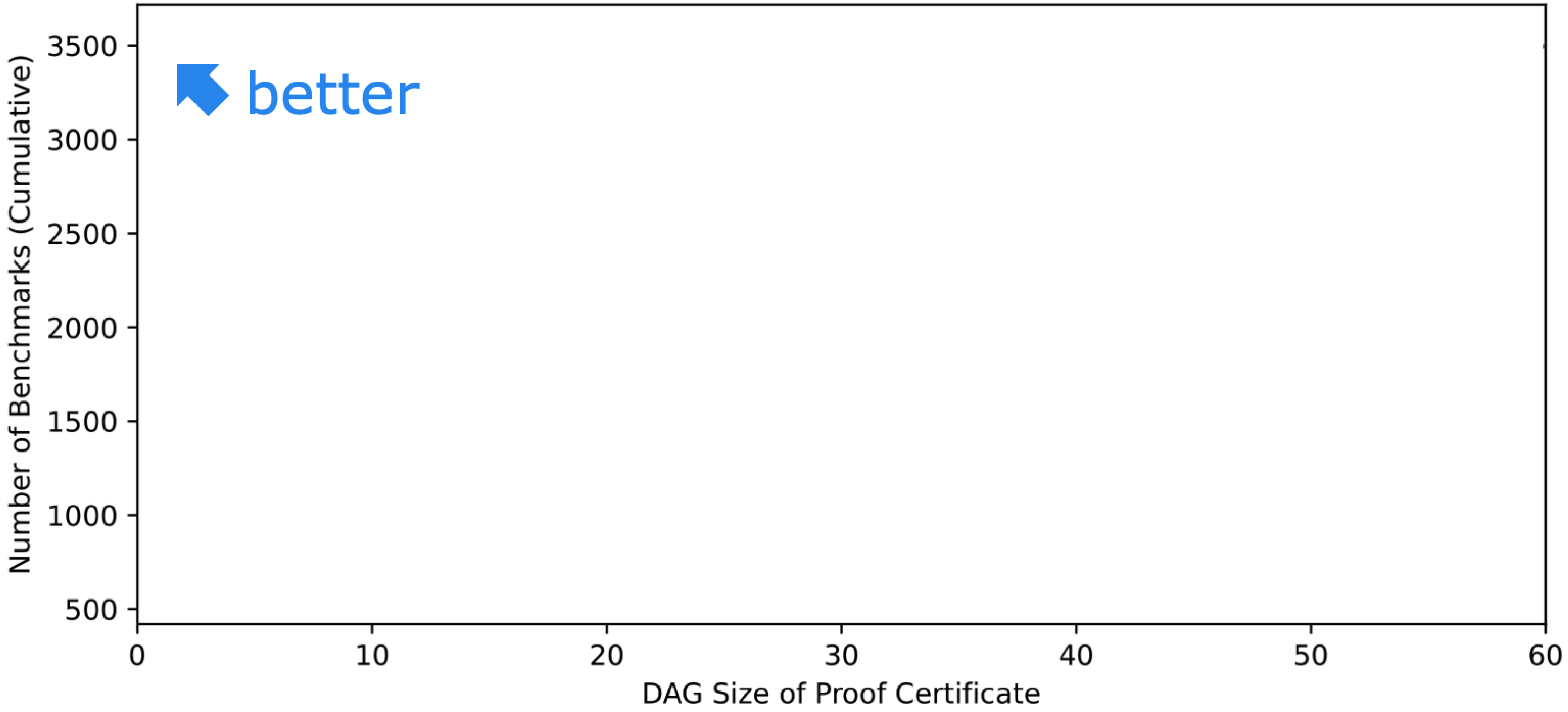


Results



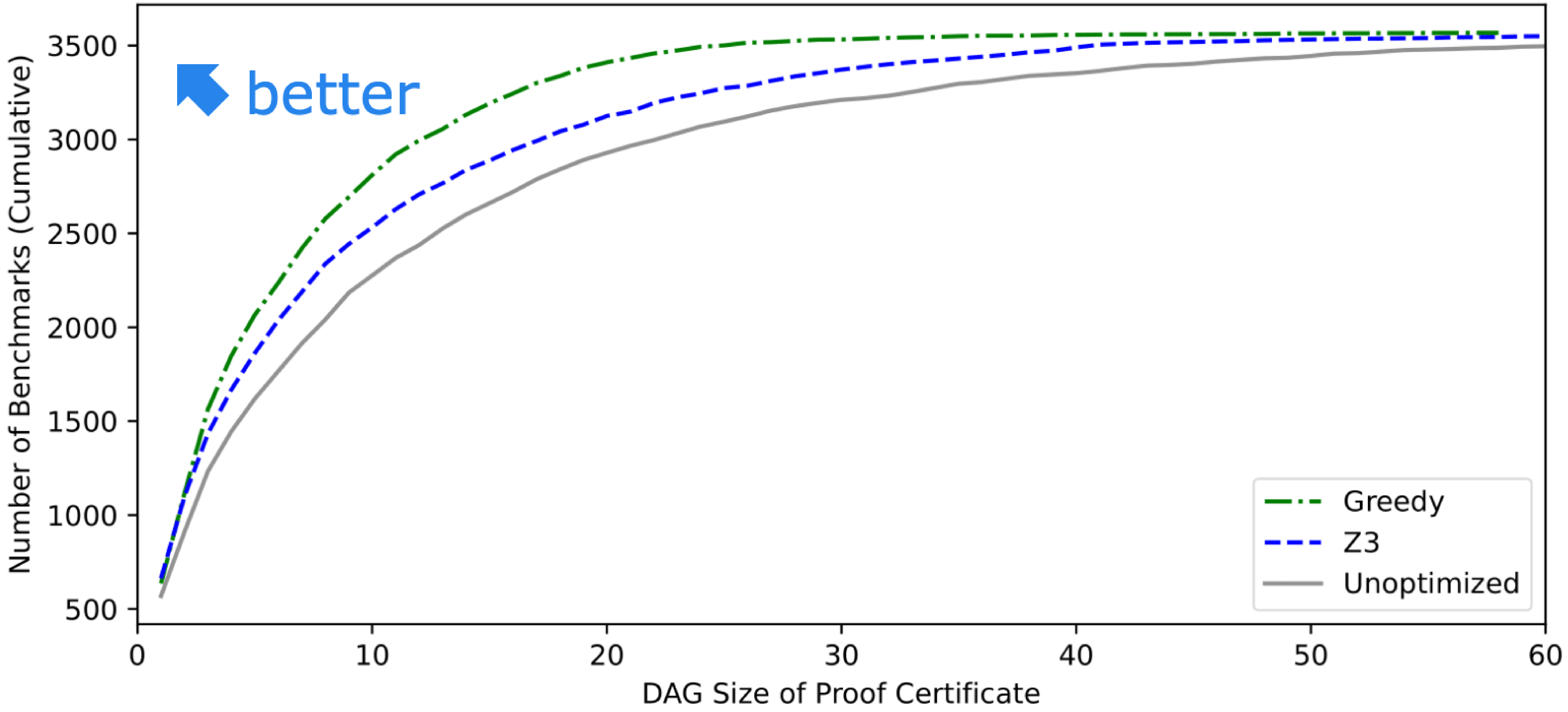
Number of Benchmarks (Cumulative)
3500
3000
2500
2000
1500
1000
500
0
10
20
30
40
50
60
DAG Size of Proof Certificate

Results



better
Number of Benchmarks (Cumulative)
3500
3000
2500
2000
1500
1000
500
0
10
20
30
40
50
60
DAG Size of Proof Certificate

Results



better
Number of Benchmarks (Cumulative)
3500
3000
2500
2000
1500
1000
500
0
10
20
30
40
50
60
DAG Size of Proof Certificate
Greedy
Z3
Unoptimized

Intel Case Study

Multi-operation circuit optimization and translation validation with egg

Intel Case Study

Multi-operation circuit optimization and translation validation with egg

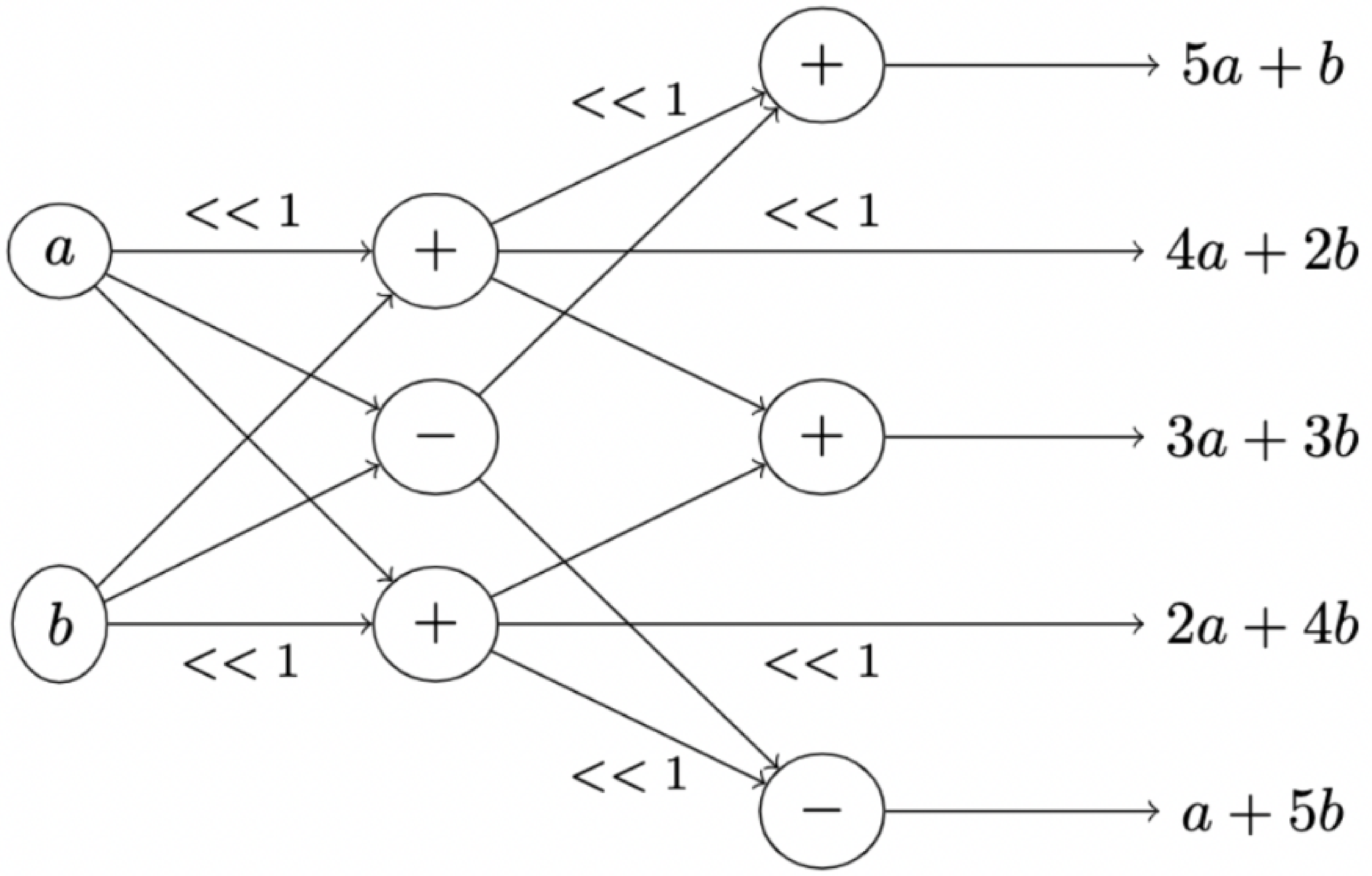
4.7 hours -> 2.3 hours

Intel Case Study

Multi-operation circuit optimization and translation validation with egg

4.7 hours -> 2.3 hours

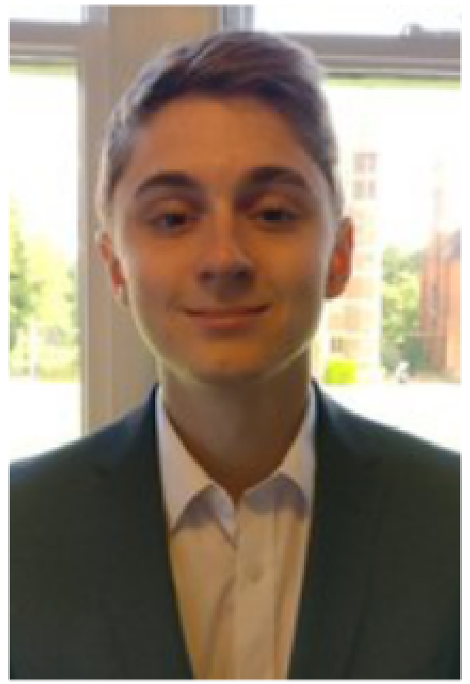
2 inputs



5 outputs

Team and Acknowledgments


Oliver Flatt


Samuel Coward


Max Willsey


Zachary Tatlock


Pavel Panchekha

Special thanks to:

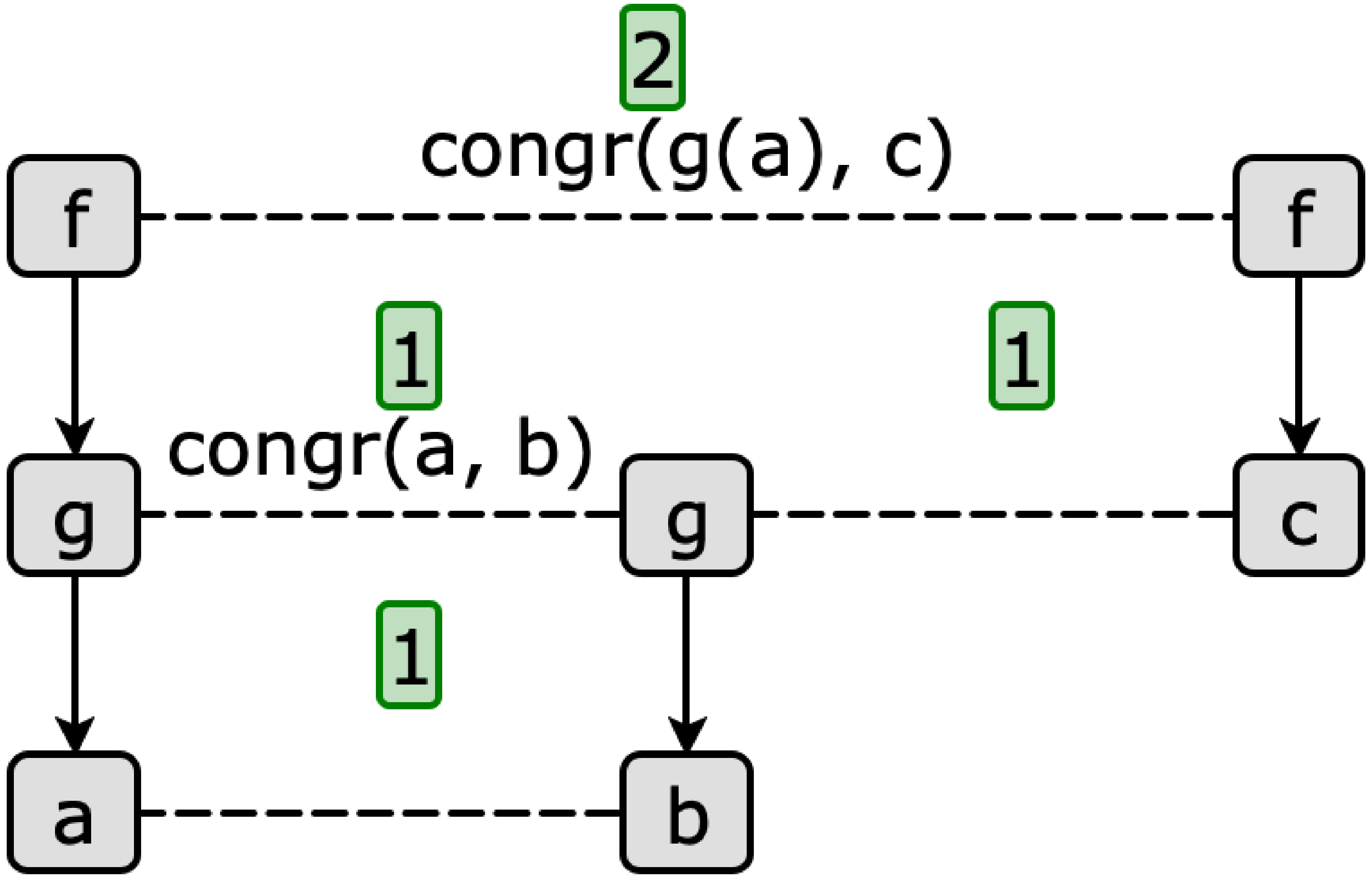
Theo Drane (Intel)

George A. Constantinides (Imperial College)

Leonardo de Moura (Microsoft)

Questions?

1. Compute size estimates
2. Find shortest path
3. Output extracted proof



oflatt@cs.washington.edu