Verified Compilation and Optimization of Floating-Point Programs in CakeML

Heiko Becker, Robert Rabe, Eva Darulova, Magnus O. Myreen, Zachary Tatlock, Ramana Kumar, Yong Kiam Tan, Anthony Fox

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Floating-Point Arithmetic in Unverified & Verified Compilers

Floating-Point Arithmetic in Unverified & Verified Compilers

COMPCERT

- IEEE-754 arithmetic
- no performance optimizations
- full correctness proof

Floating-Point Arithmetic in Unverified & Verified Compilers

LLVM

- IEEE-754 arithmetic
- fast-math optimizations
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CAKEML
A Verified Implementation of ML

- no floating-point support

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Floating-Point Arithmetic in Unverified & Verified Compilers

GCC

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before our work

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Floating-Point Arithmetic in Unverified & Verified Compilers

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in this talk

CAKEML
A Verified Implementation of ML

- **IEEE-754 arithmetic**
- **fast-math-style optimizations**
- **correctness & accuracy proofs**

COMPCERT

- IEEE-754 arithmetic
- no performance optimizations
- full correctness proof

Fast-Math-Style Optimizations in Compilers

LLVM

$$x * (x * (x * x)) \rightarrow (x * x) * (x * x)$$

Fast-Math-Style Optimizations in Compilers

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changes bit-level result

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CAKEML

A Verified Implementation of ML

preserve IEEE-754 floating-point arithmetic

Fast-Math-Style Optimizations in Compilers

LLVM

$x * (x * (x * x)) \rightarrow (x * x) * (x * x)$

changes bit-level result

preserve IEEE-754 floating-point arithmetic

requires bit-level accuracy

Fast-Math-Style Optimizations in Compilers

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CAKEML
A Verified Implementation of ML

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Fast-Math-Style Optimizations in Compilers

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accuracy

CAV'19

Icing: Supporting Fast-math Style Optimizations in a Verified Compiler

Heiko Becker[1], Eva Darulova[1], Magnus O. Myreen[2], and Zachary Tatlock[3]*

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[3] University of Washington, ztatlock@cs.washington.edu

Abstract. Verified compilers like CompCert and CakeML offer increasingly sophisticated optimizations. However, their deterministic source

Icing: Supporting Fast-math Style Optimizations in a Verified Compiler [CAV'19]

verified floating-point optimizations

Icing: Supporting Fast-math Style Optimizations in a Verified Compiler [CAV'19]

proof-of-concept optimizer

verified floating-point optimizations

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proof-of-concept optimizer

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fine-grained control

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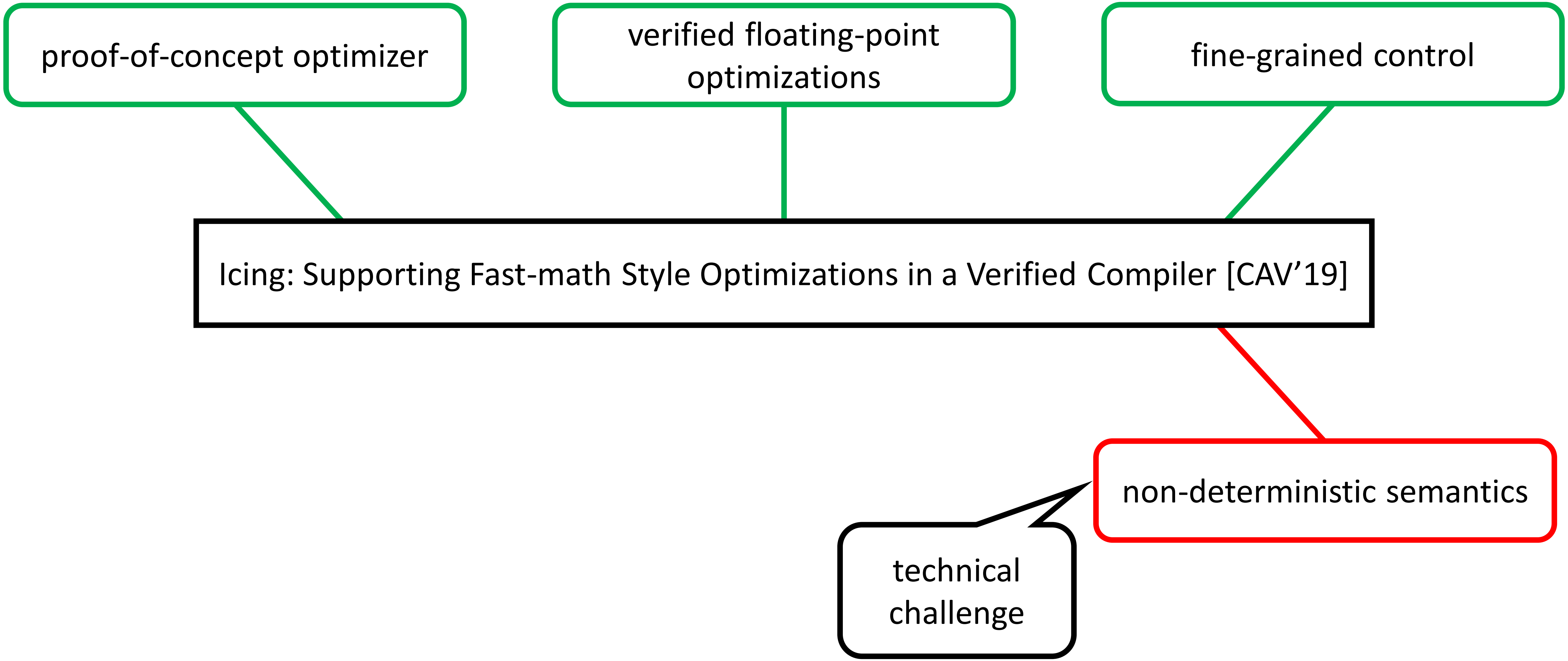
Icing: Supporting Fast-math Style Optimizations in a Verified Compiler [CAV'19]

non-deterministic semantics


proof-of-concept optimizer
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Icing: Supporting Fast-math Style Optimizations in a Verified Compiler [CAV'19]
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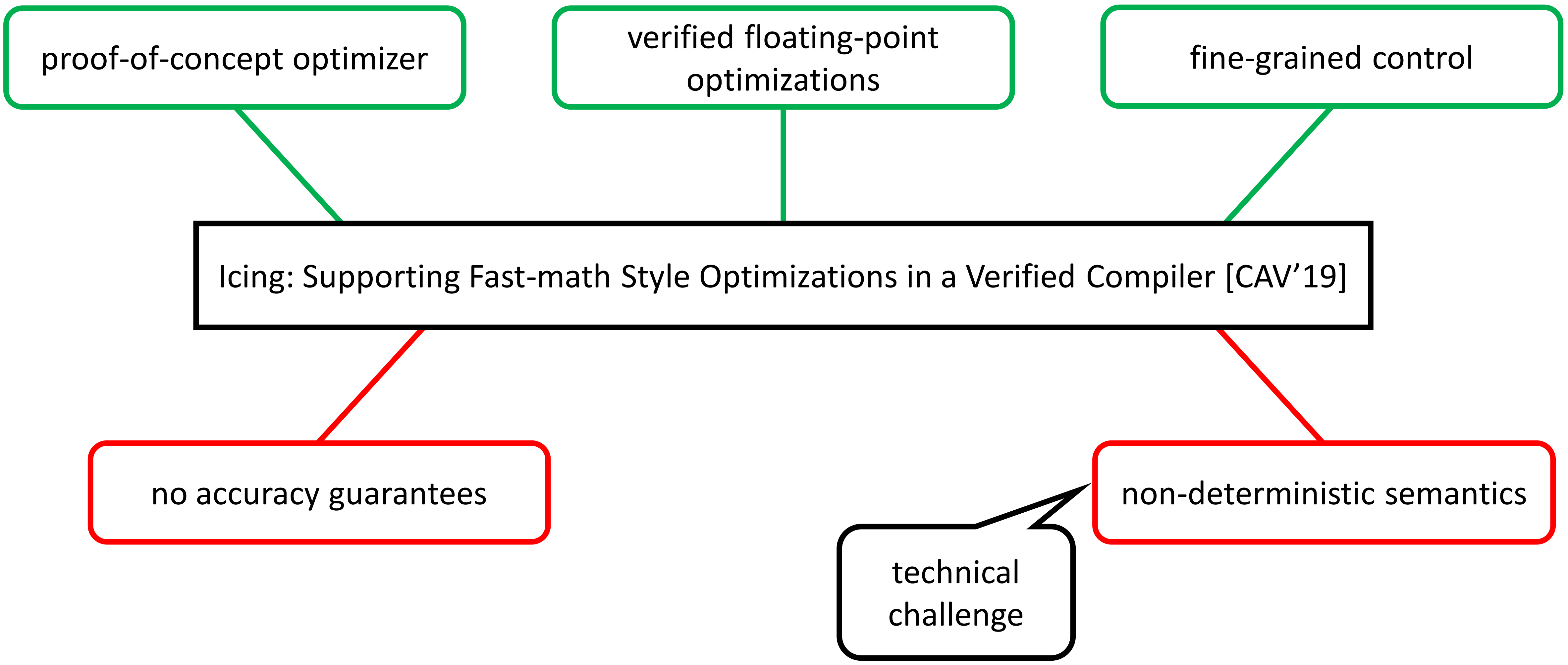
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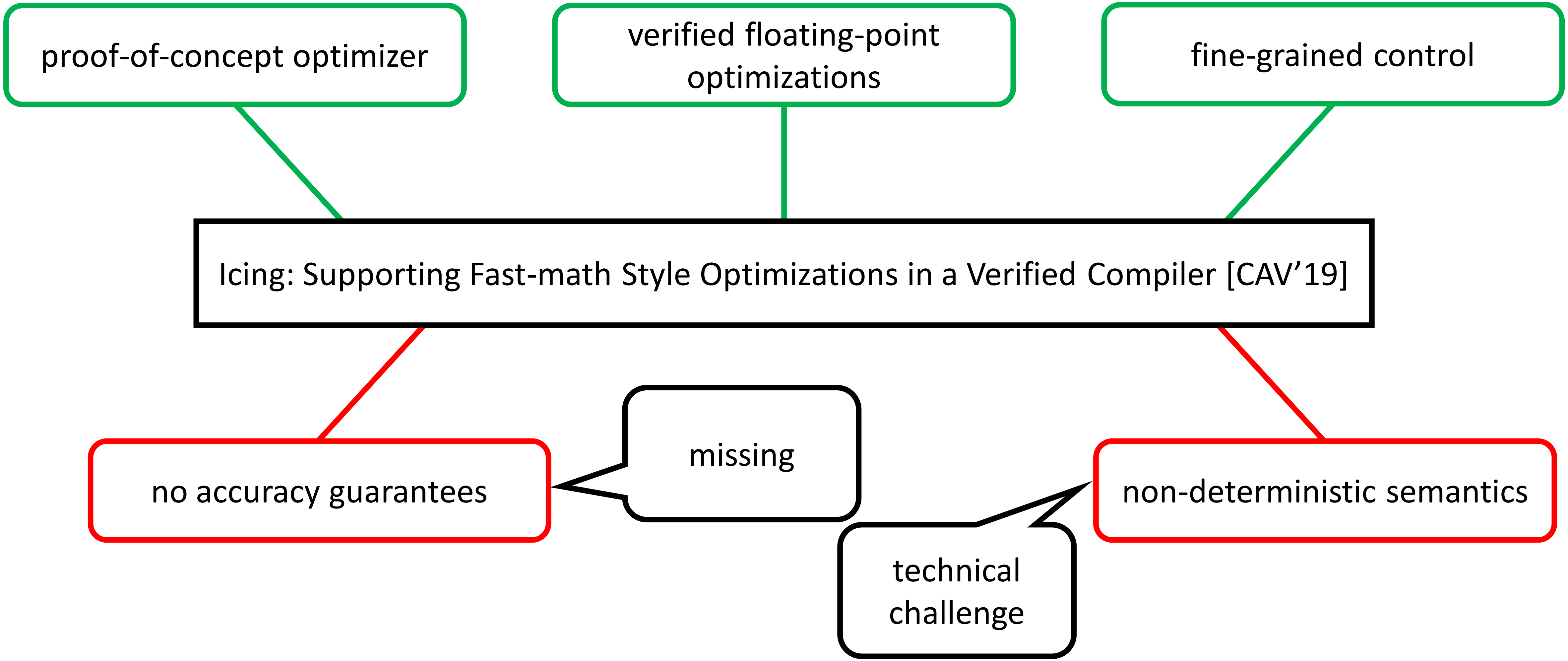
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Icing: Supporting Fast-math Style Optimizations in a Verified Compiler [CAV'19]
no accuracy guarantees
missing
technical challenge
non-deterministic semantics

Why Accuracy Matters

```
(* require(1.0 ≤ x ≤ 100.0 ∧
          1.0 ≤  y ≤100.0)
                 *)
fun cartToPol_x (x:double, y:double):double =
  sqrt((x * x) + (y * y))
```

Why Accuracy Matters

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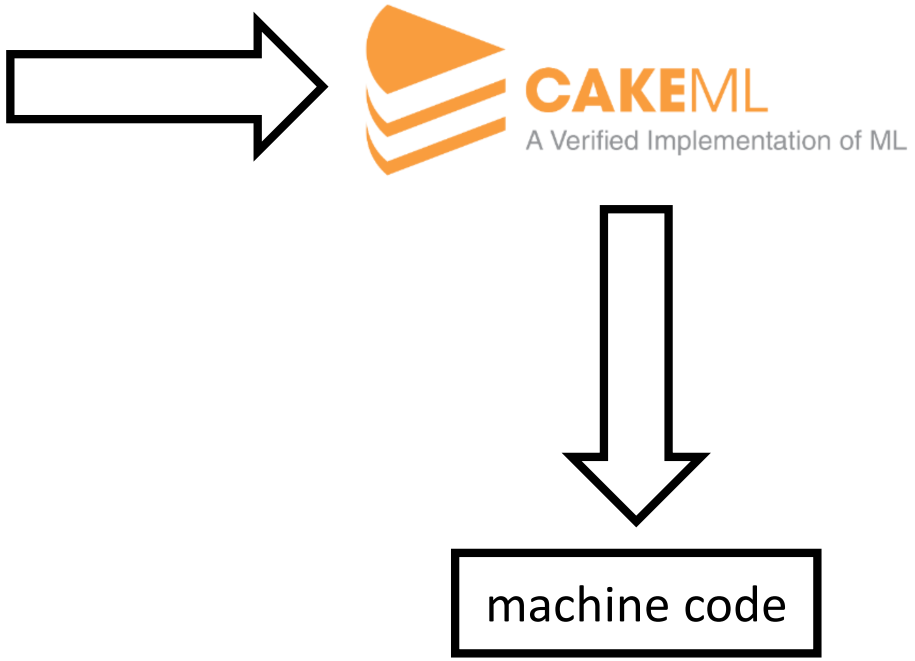

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CAKEML
A Verified Implementation of ML

IEEE-754

machine code

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program designed for unavoidable input and output errors

CAKEML
A Verified Implementation of ML

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roundoff error ≤
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machine code

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machine code | machine code | machine code

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A Verified Implementation of ML

roundoff error ≤
output error

machine code

machine code

machine code

error refinement
any optimized implementation
below output error is fine

Contributions

RealCake:

- extends CakeML with **relaxed non-deterministic floating-point semantics**
- optimizes with a **fast-math optimizer**
- **soundly proves roundoff errors** of floating-point kernels with automated tools
- proves **error refinement**

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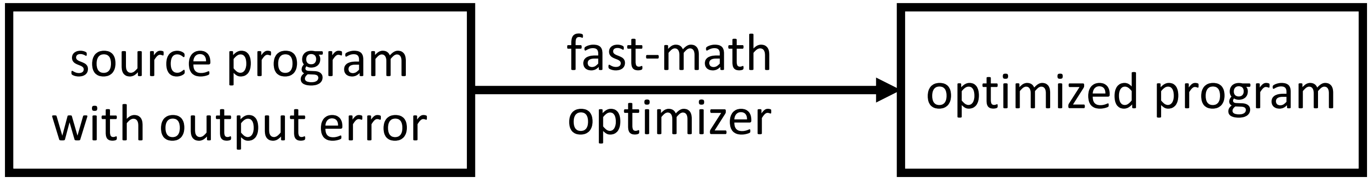
The RealCake Compiler Zoomed In

source program
with output error

The RealCake Compiler Zoomed In



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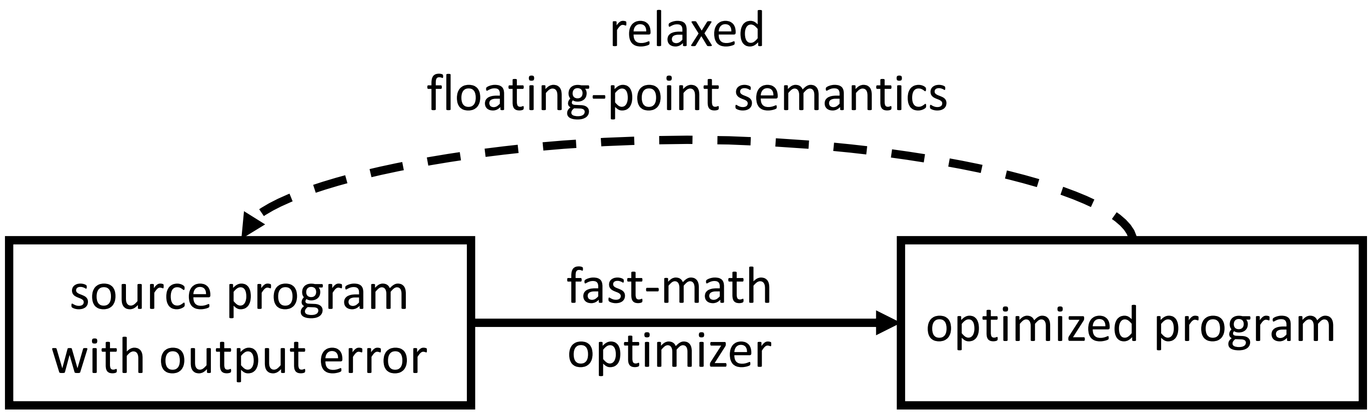

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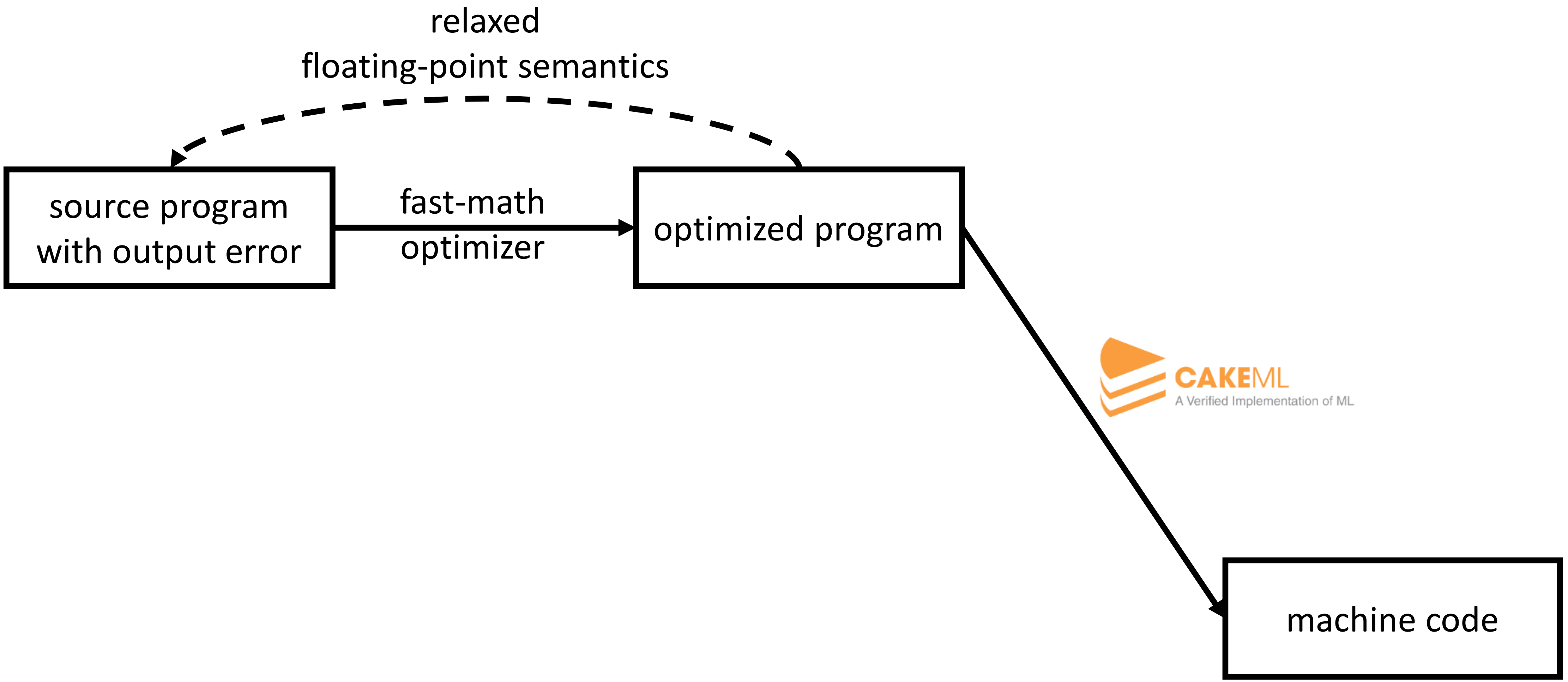

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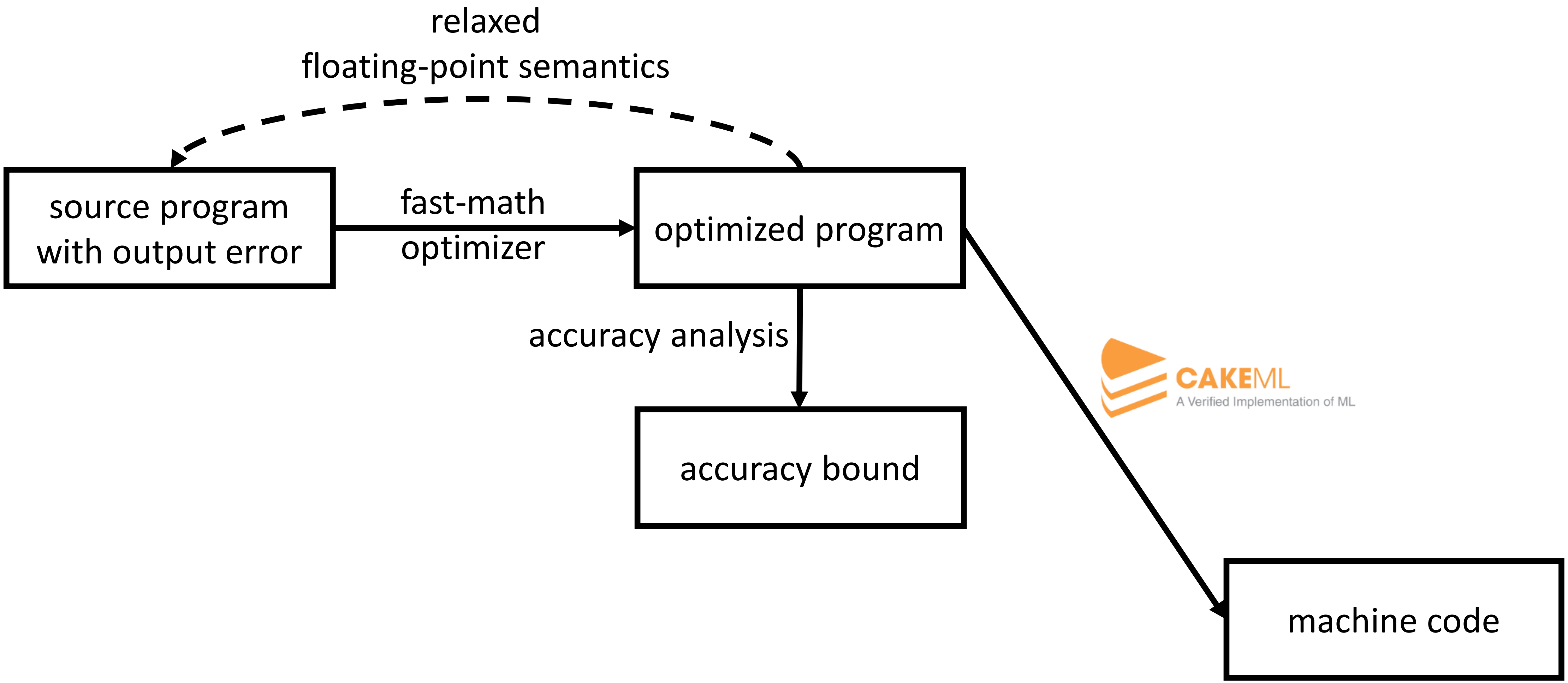

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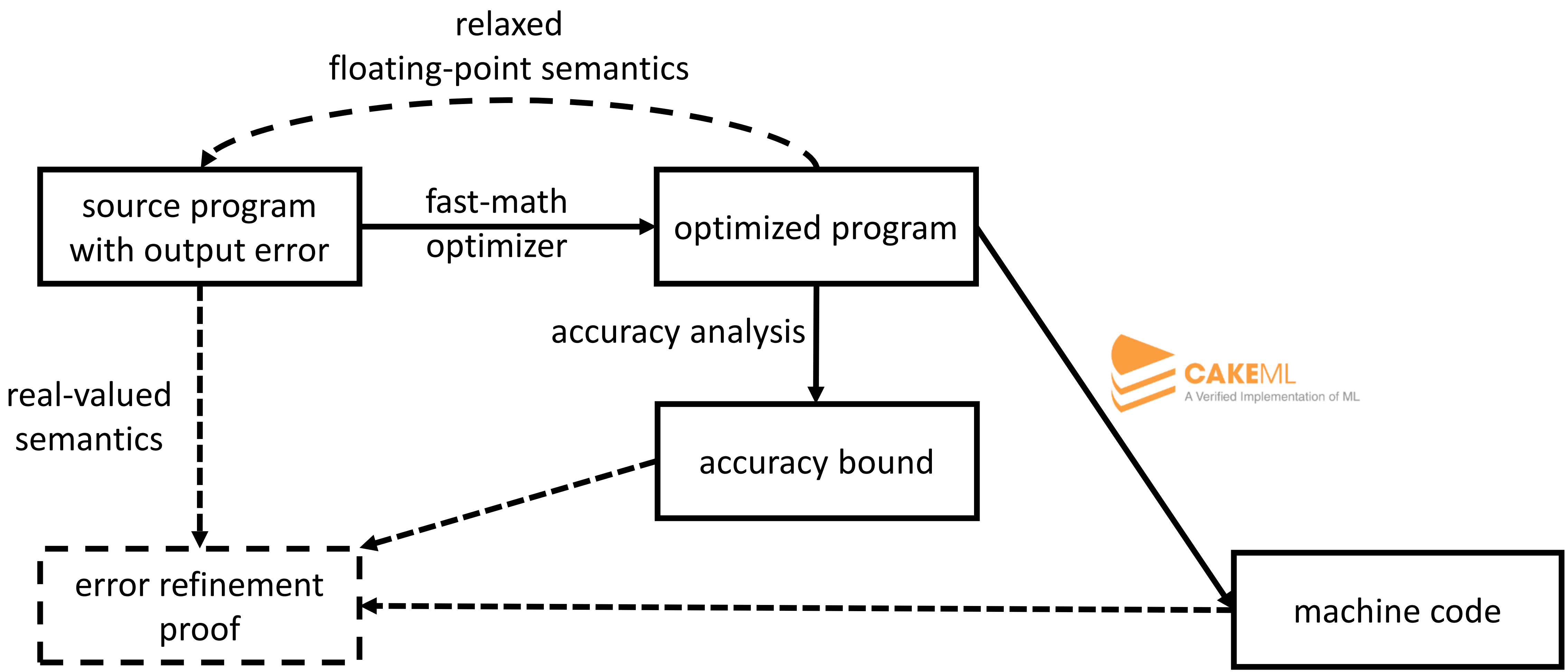

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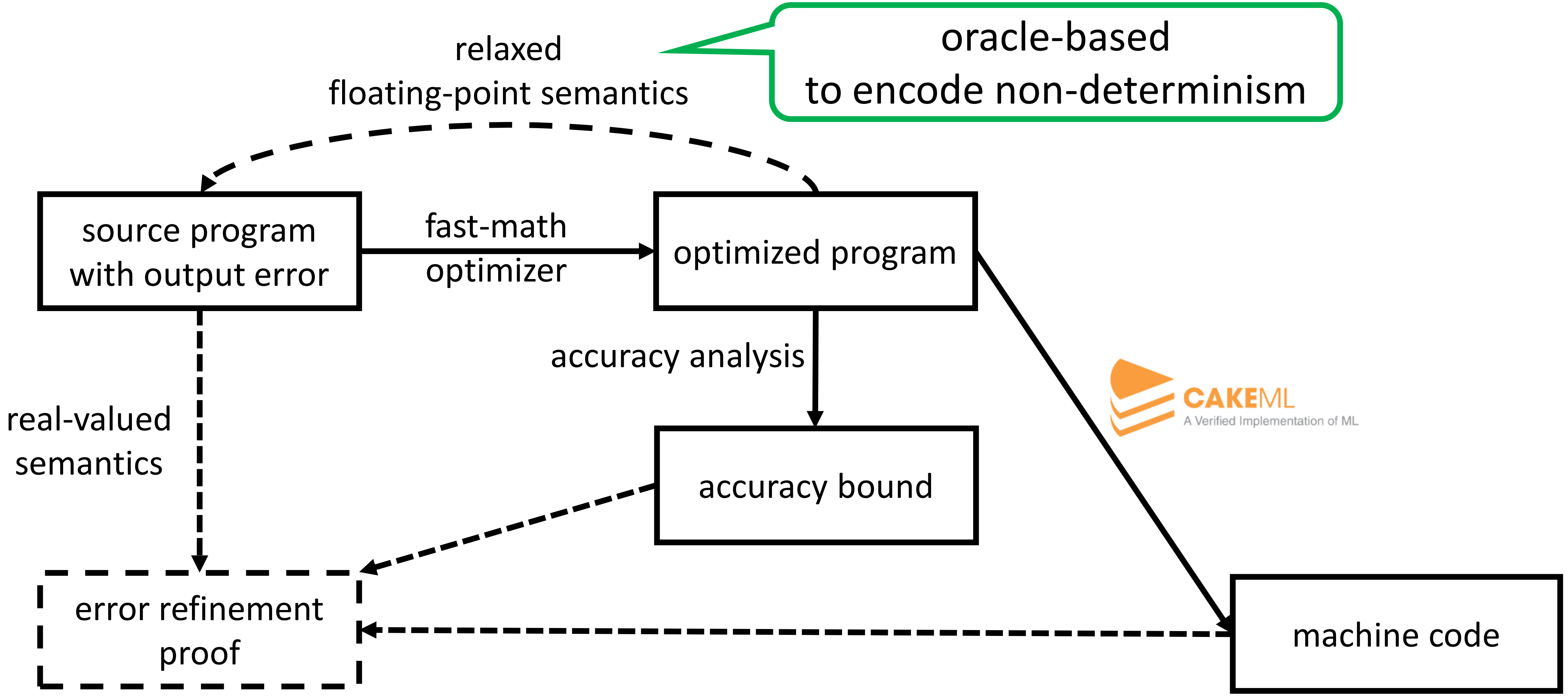

Floating-Point Programs in CakeML

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fun jetEngine(x1:double, x2:double):double =
  opt: (let val t = (((3.0 * x1) * x1) + (2.0 * x2)) - x1
    val t2 = (((3.0 * x1) * x1) - (2.0 * x2)) - x1
    val d = (x1 * x1) + 1.0
    val s = t / d
    val s2 = t2 / d
  in
    x1 + (((((((((2.0 * x1) * s) * (s - 3.0)) +
    ((x1 * x1) * ((4.0 * s) - 6.0))) * d) +
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end)
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double
operations

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```

optimization annotation

double operations

Floating-Point Programs in CakeML

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tolerable noise

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input constraints

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tolerable noise

Optimized Floating-Point Programs

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(* guaranteed error bound: 2^-5,
precondition P: 0.0 ≤ x1 ≤ 5.0 ∧ −20.0 ≤ x2 ≤ 5.0 *)
fun jetEngine(x1:double, x2:double):double =
  noopt: (let
    val t = fma((x1+x1)+x1, x1, (x2 + x2) - x1)
    val t2 = fma((x1+x1)+x1, x1, fma(-2.0, x2, -x1))
    val d = fma(x1, x1, 1.0)
    val s = t / d
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  in
    x1 + fma(x1 * d, fma((s - 3.0) + (s - 3.0), s,
      x1 * fma(4.0, s, -6.0)),
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verified by accuracy analysis

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verified by accuracy analysis

faster, locally more accurate

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verified by accuracy analysis

faster, locally more accurate

disallow further optimization

Optimized Floating-Point Programs

machine code from IEEE-754 preserving compilation in CakeML

disallow further optimization

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verified by accuracy analysis

faster, locally more accurate

The Final Specification Theorem

$$
\begin{array}{l}
\texttt{jetEngineInputsInPrecond}\ (s_1, s_2)\ (w_1, w_2)\ \wedge\ \texttt{environmentOk}\ ([\mathit{jetEngine};\ s_1;\ s_2], \mathit{fs})\ \Rightarrow \\
\exists\, w\ r. \\
\quad \texttt{CakeMLevaluatesAndPrints}\ (\mathit{jetEngineCode}, s_1, s_2, \mathit{fs})\ (\texttt{toString}\ w)\ \wedge \\
\quad \texttt{initialFPcodeReturns}\ \mathit{jetEngineUnopt}\ (w_1, w_2)\ w\ \wedge \\
\quad \texttt{realSemanticsReturns}\ \mathit{jetEngineUnopt}\ (w_1, w_2)\ r\ \wedge\ \texttt{abs}\ (\texttt{fpToReal}\ w\ -\ r)\ \leq\ 2^{-5}
\end{array}
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The Final Specification Theorem

inputs in
specified
constraints

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The Final Specification Theorem

inputs in specified constraints

the program is run with the correct inputs

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The Final Specification Theorem

inputs in specified constraints

program returns double word w

the program is run with the correct inputs

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w also result of nondeter ministic semantics

The Final Specification Theorem

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\texttt{jetEngineInputsInPrecond}\ (s_1, s_2)\ (w_1, w_2)\ \wedge\ \texttt{environmentOk}\ ([\mathit{jetEngine};\ s_1;\ s_2], \mathit{fs})\ \Rightarrow \\
\exists\, w\ r. \\
\quad \texttt{CakeMLevaluatesAndPrints}\ (\mathit{jetEngineCode}, s_1, s_2, \mathit{fs})\ (\texttt{toString}\ w)\ \wedge \\
\quad \texttt{initialFPcodeReturns}\ \mathit{jetEngineUnopt}\ (w_1, w_2)\ w\ \wedge \\
\quad \texttt{realSemanticsReturns}\ \mathit{jetEngineUnopt}\ (w_1, w_2)\ r\ \wedge\ \texttt{abs}\ (\texttt{fpToReal}\ w\ -\ r)\ \leq\ 2^{-5}
\end{array}
$$

w also result of nondeter ministic semantics

real-number semantics returns r

The Final Specification Theorem

inputs in specified constraints

program returns double word w

the program is run with the correct inputs

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`jetEngineInputsInPrec` ... $tEngine;\ s_1;\ s_2], fs) \Rightarrow$

$\exists\, w\ r.$

`CakeMLevaluatesAndP` ... $w) \wedge$

`initialFPcodeReturn` ...

`realSemanticsReturn` ... `Real` $w - r) \leq 2^{-5}$

RealCake is the first verified compiler that proves end-to-end accuracy bounds for **compiled** fast-math optimized programs

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output error sound

RealCake’s Four-Phase-Optimizer

canonical form

RealCake's Four-Phase-Optimizer

canonical form

$$x - y \rightarrow x + ((-1) \times y)$$
$$((x + y) + z \rightarrow x + (y + z)$$

move constants to right-hand sides

RealCake’s Four-Phase-Optimizer

canonical form → undistribute

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RealCake's Four-Phase-Optimizer

canonical form → undistribute

$(x \times y) + (x \times z) \rightarrow x \times (y + z)$

$x - y \rightarrow x + ((-1) \times y)$

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RealCake's Four-Phase-Optimizer

canonical form → undistribute → peephole optimizer

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$(x \times y) + (x \times z) \to x \times (y + z)$

$x \times 0 \to 0$
$x \times 1 \to x$
$x \times -1 \to -x$
$x \times 2 \to x + x$
$x \times 3 \to x + (x + x)$
$x * y + z \to \mathrm{fma}(x, y, z)$ …

RealCake's Four-Phase-Optimizer

canonical form → undistribute → peephole optimizer → balance trees

$$x - y \rightarrow x + ((-1) \times y)$$
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RealCake's Four-Phase-Optimizer

canonical form → undistribute → peephole optimizer → balance trees

canonical form:

$$x - y \to x + ((-1) \times y)$$
$$((x + y) + z \to x + (y + z)$$

move constants to right-hand sides

undistribute:

$$(x \times y) + (x \times z) \to x \times (y + z)$$

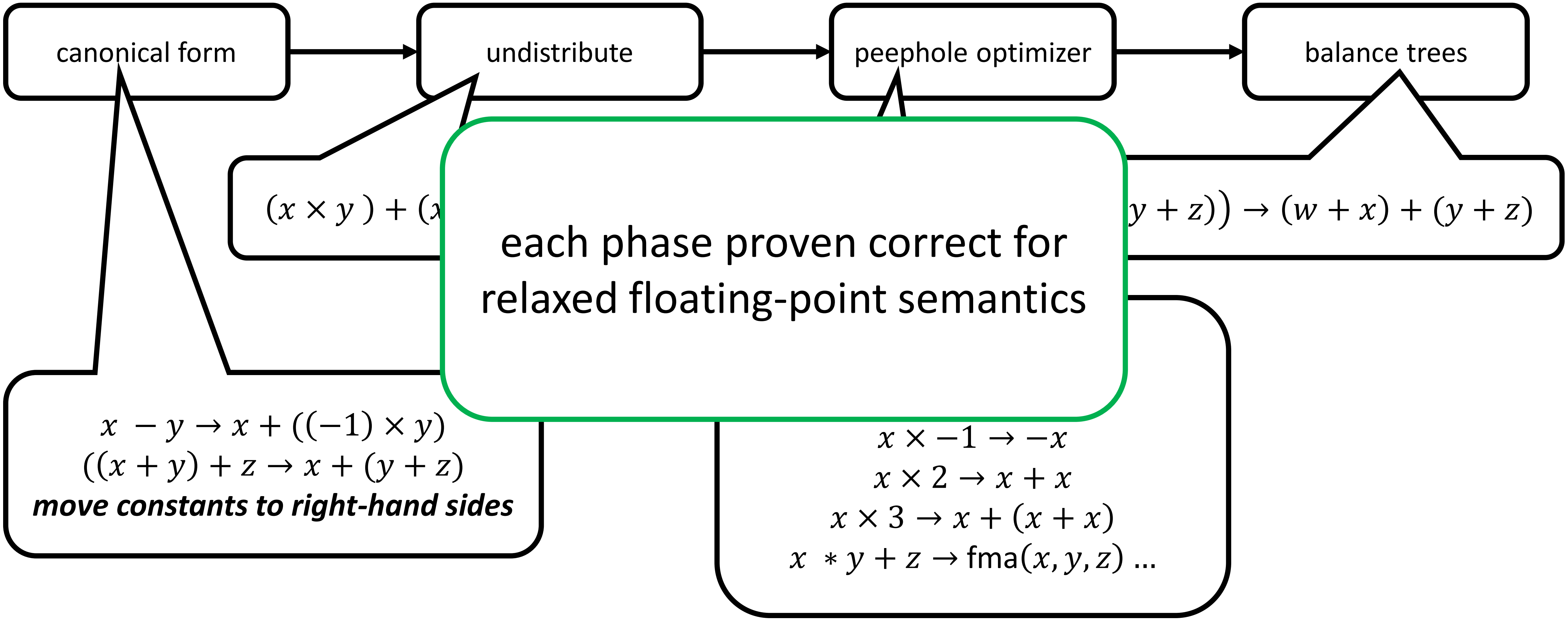
peephole optimizer:

$$x \times 0 \to 0$$
$$x \times 1 \to x$$
$$x \times -1 \to -x$$
$$x \times 2 \to x + x$$
$$x \times 3 \to x + (x + x)$$
$$x * y + z \to \mathrm{fma}(x, y, z) \ldots$$

balance trees:

$$w + \big(x + (y + z)\big) \to (w + x) + (y + z)$$

RealCake's Four-Phase-Optimizer



Experimental Results – Roundoff Errors

Name	
cartToPol	
delta	
doppler2	
pid	
sine_newton	
sqroot	
turbine1	

Experimental Results – Roundoff Errors

Name	Original
cartToPol	2.815e-09
delta	1.970e-13
doppler2	6.534e-13
pid	7.621e-15
sine_newton	7.495e-15
sqroot	1.115e-15
turbine1	1.588e-13

Experimental Results – Roundoff Errors

Name	Original	fast-math
cartToPol	2.815e-09	2.463e-09
delta	1.970e-13	2.940e-12
doppler2	6.534e-13	1.639e-12
pid	7.621e-15	7.727e-15
sine_newton	7.495e-15	6.27e-15
sqroot	1.115e-15	1.059e-15
turbine1	1.588e-13	1.541e-13

Experimental Results – Roundoff Errors

Name	Original	fast-math	Improvement
cartToPol	2.815e-09	2.463e-09	13%
delta	1.970e-13	2.940e-12	-198%
doppler2	6.534e-13	1.639e-12	50%
pid	7.621e-15	7.727e-15	-1%
sine_newton	7.495e-15	6.27e-15	16%
sqroot	1.115e-15	1.059e-15	5%
turbine1	1.588e-13	1.541e-13	3%

Experimental Results – Roundoff Errors

Name	Original	fast-math	Improvement
cartToPol	2.815e-09	2.463e-09	13%
delta	1.970e-13	2.940e-12	-198%
doppler2	6.534e-13	1.639e-12	50%
pid	7.621e-15	7.727e-15	-1%
sine_newton	7.495e-15	6.27e-15	16%
sqroot	1.115e-15	1.059e-15	5%
turbine1	1.588e-13	1.541e-13	3%

trade accuracy for performance

Experimental Results – Performance

Name
cartToPol
delta
doppler2
pid
sine_newton
sqroot
turbine1

Experimental Results – Performance

Name	Original
cartToPol	2.05
delta	13.49
doppler2	36.00
pid	104.11
sine_newton	126.34
sqroot	87.06
turbine1	121.02

Experimental Results – Performance

Name	Original	fast-math
cartToPol	2.05	9%
delta	13.49	16%
doppler2	36.00	6%
pid	104.11	0%
sine_newton	126.34	0%
sqroot	87.06	5%
turbine1	121.02	0%

Experimental Results – Performance

Name	Original	Csts	fast-math
cartToPol	2.05	1%	9%
delta	13.49	1%	16%
doppler2	36.00	91%	6%
pid	104.11	96%	0%
sine_newton	126.34	92%	0%
sqroot	87.06	95%	5%
turbine1	121.02	96%	0%

Conclusion

RealCake:

- proves **error refinement for CakeML programs**
- extends CakeML with **oracle-based floating-point semantics**
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Conclusion

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in the paper:

- verified constant lifting optimization
- heuristic to avoid slow-downs
- integration into CakeML toolchain
- implementation of real-numbered and IEEE-754 semantics